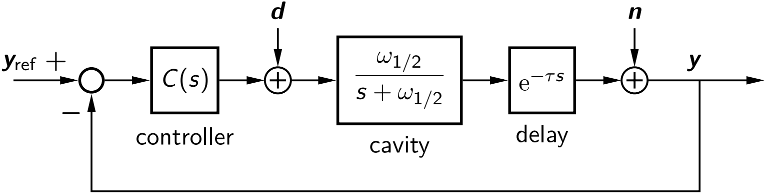
Towards a better understanding of cavity field control

Olof Troeng, Bo Bernhardsson, Anders J Johansson (Lund U.)
LLRF17, 2017-10-19

Cavity Field Control



Some perspectives on the classic field control problem:
(coming from automatic control, high-intensity proton linac)

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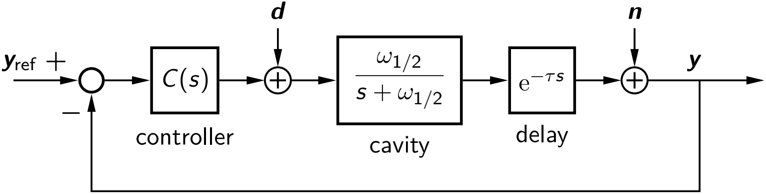


Some perspectives on the classic field control problem:
(coming from automatic control, high-intensity proton linac)

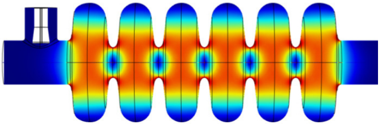

Cavity modeling, normalization

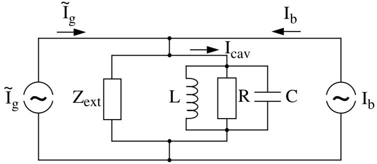

Phasor diagrams, directionality


Complex-coefficient systems


Parasitic modes

Accelerator Cavity Modeling (1/2)

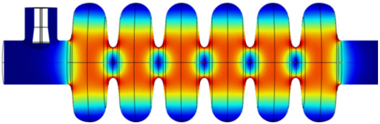




$$\frac{d\mathbf{V}}{dt} = (-\omega_{1/2} + i\Delta\omega)\mathbf{V} + R_L\omega_{1/2}\left(2\mathbf{I_g} + \mathbf{I_b}\right)$$

$$P_g = \frac{1}{4}\frac{r}{Q}Q_{\text{ext}}\left|\mathbf{I_g}\right|^2$$

Accelerator Cavity Modeling (1/2)

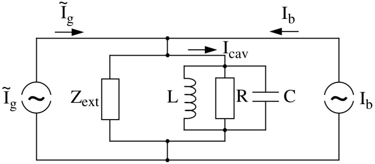


$$\frac{d\mathbf{A}}{dt} = (-\gamma + i\Delta\omega)\mathbf{A} + \sqrt{2\gamma_{\mathrm{ext}}}\mathbf{F_g} + \frac{\alpha}{2}\mathbf{I_b}$$

A – Mode amplitude $[\sqrt{\mathrm{J}}]$

$\mathbf{F_g}$ – Forward wave $[\sqrt{\mathrm{W}}]$

[Haus 1983]



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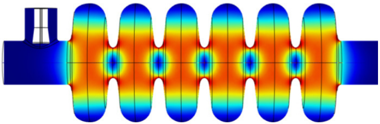

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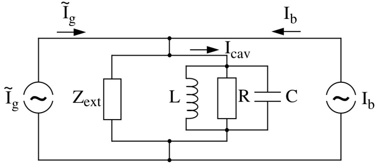
[Haus 1983]

$\mathbf{A}$ – Mode amplitude $[\sqrt{\mathrm{J}}]$

$\mathbf{F_g}$ – Forward wave $[\sqrt{\mathrm{W}}]$

$\mathbf{V} = \alpha\mathbf{A} \qquad \left(\alpha = \sqrt{\omega_a(r/Q)}\right)$

$P_g = |\mathbf{F_g}|^2$



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Accelerator Cavity Modeling (2/2)

Although we will soon normalize the model, the proposed parametrization [Waves & Field in Optoelectronics, Haus (1984)] has the following advantages:

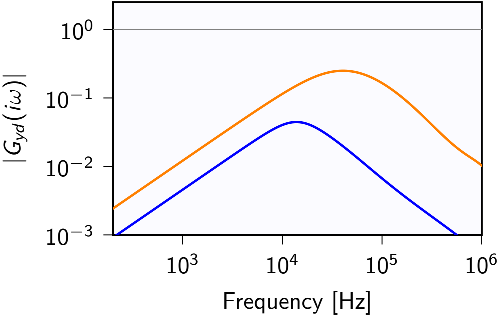
- Cleaner expressions, e.g., $P_g = |\mathbf{F_g}|^2$
- Mode states are not dependent on the particle velocity
- Cleaner treatment of parasitic modes of elliptical cavities
- Allows more direct derivation from Maxwell's eqs.

Normalization

Normalization of accelerating mode dynamics around nominal operating point gives:

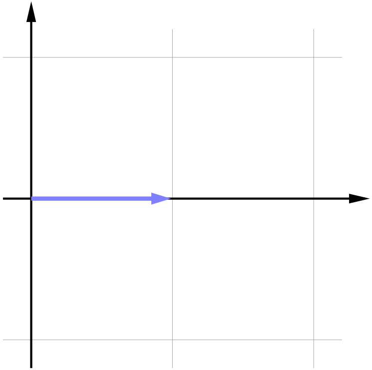
$$Y(s) = \underbrace{\frac{\gamma}{s + \gamma - i\Delta\omega}}_{P_{\text{cav}}(s)} \left[U(s) + K_g D_g(s) + K_b D_b(s) \right]$$

- K_g, K_b – dimensionless constants,
  typically $1 \le |K_g| \le 2$, $0 \le |K_b| \le 1$

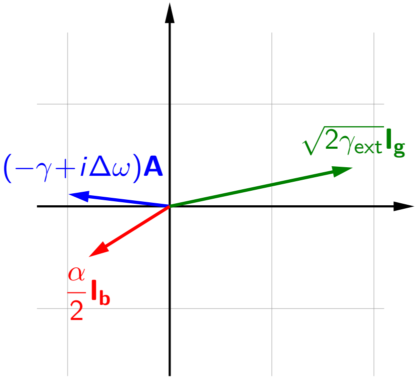


Phasor Diagrams

$$\frac{d\mathbf{A}}{dt} = (-\gamma + i\Delta\omega)\mathbf{A} + \sqrt{2\gamma_{\text{ext}}}\mathbf{I_g} + \frac{\alpha}{2}\mathbf{I_b}$$



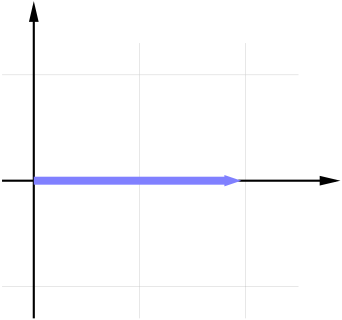
Mode amplitude $\mathbf{A}$ $[\sqrt{\text{J}}]$

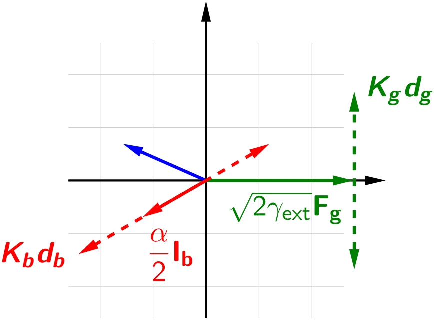


Terms of $\frac{d}{dt}\mathbf{A}$ $[\sqrt{\text{J}}/\text{s}]$

Directionality in the Control Problem

$$\frac{d\mathbf{A}}{dt} = (-\gamma + i\Delta\omega)\mathbf{A} + \sqrt{2\gamma_{\text{ext}}}\mathbf{I_g} + \frac{\alpha}{2}\mathbf{I_b}$$





Directionality of disturbances

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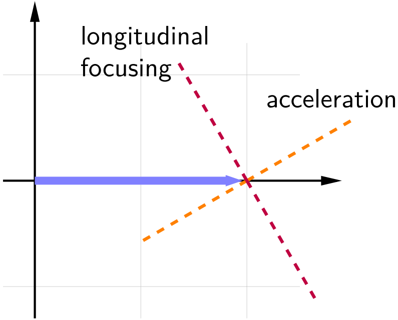
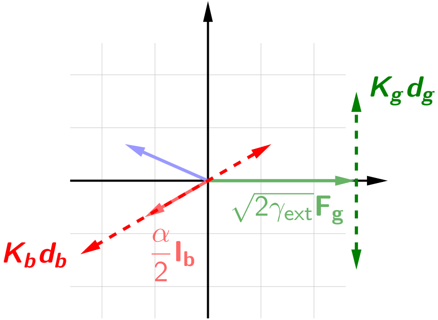


Directionality of objective



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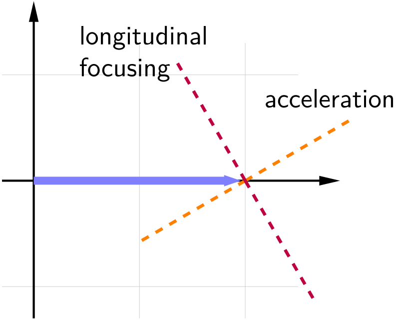

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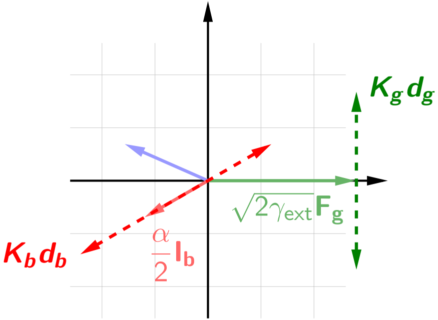
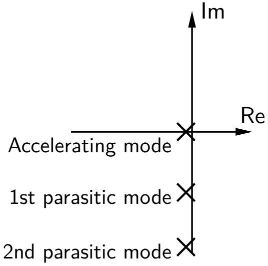


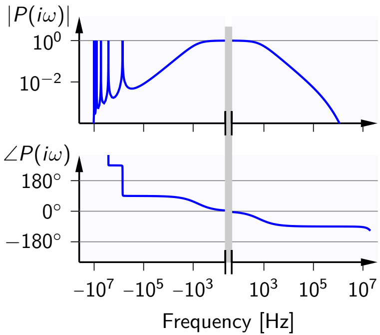
Directionality of disturbances

Implications:

- Actual performance not directly dependent on A and ϕ
- Optimal controller is not rotationally symmetric

Control Theory for Complex-Coefficient Systems





$$G(s) = \sum_{k=a,1,\ldots} \frac{c_k \gamma_k}{s + \gamma_k - i\Delta\omega_k}$$

- Standard control theory applies $\left(\text{if } T \mapsto * \text{ and } \int_0^\infty \mapsto \int_{-\infty}^\infty\right)$, e.g., the Nyquist criterion; negative frequencies are significant

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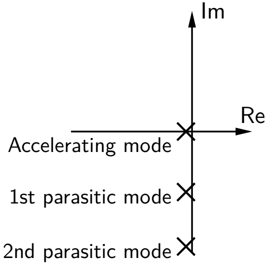

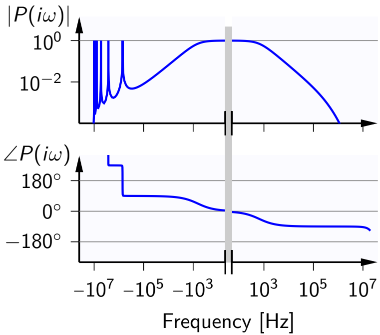

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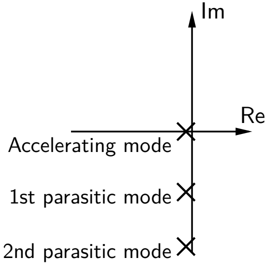

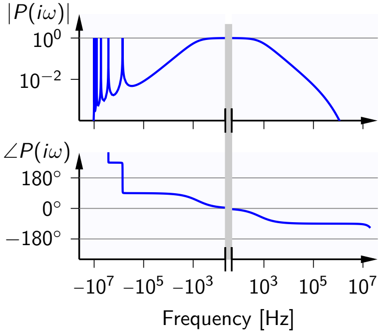

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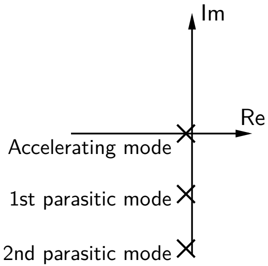

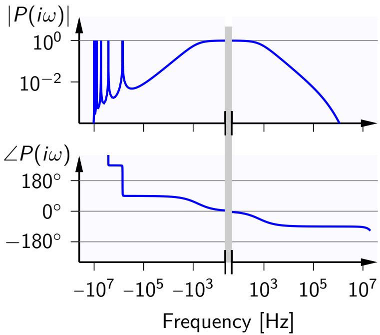

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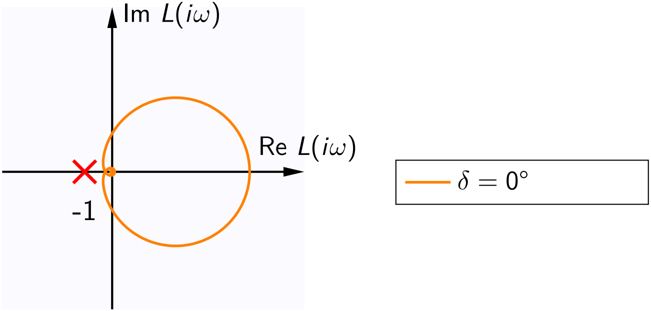
- MatLab handles complex-coefficient systems poorly
- Two useful applications: loop phase and parasitic modes

Loop Phase Adjustment

Open loop transfer function

$$L(s) = P_{\mathsf{cav}}(s)\mathrm{e}^{-sL}\mathrm{e}^{-i\theta} \cdot C_0(s)\mathrm{e}^{i\theta_{\mathsf{adj}}} = L_0(s)\mathrm{e}^{i\delta}$$

Stability and robustness depends on loop phase adjustment error $\delta = \theta_{\mathsf{adj}} - \theta$



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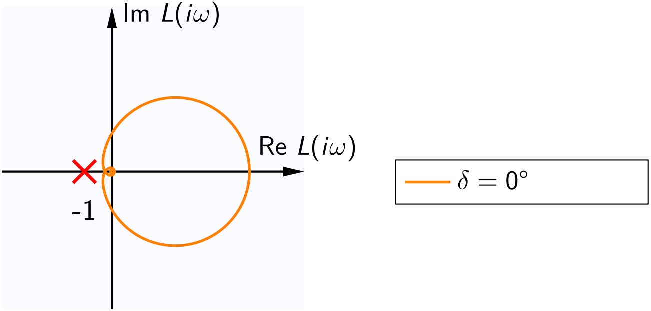

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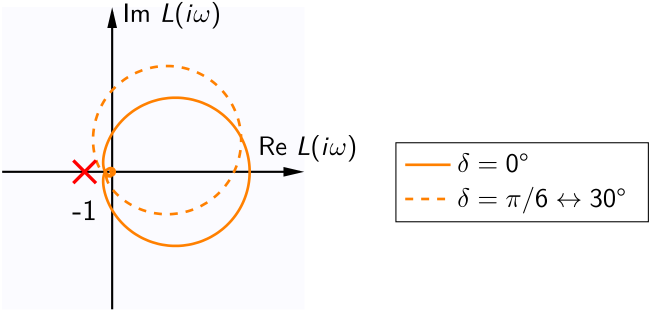

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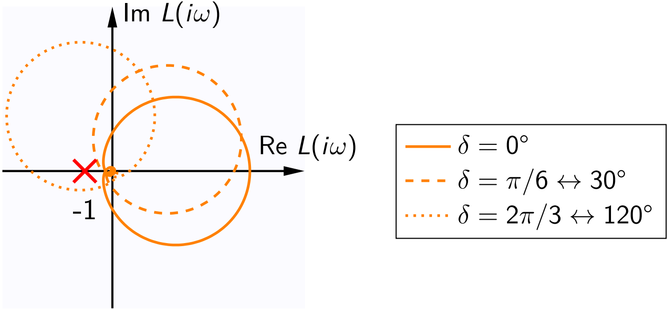
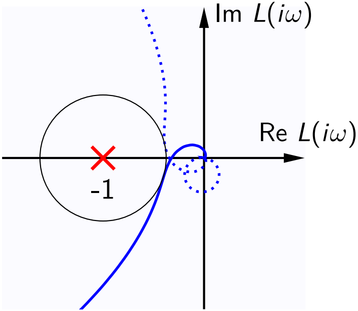
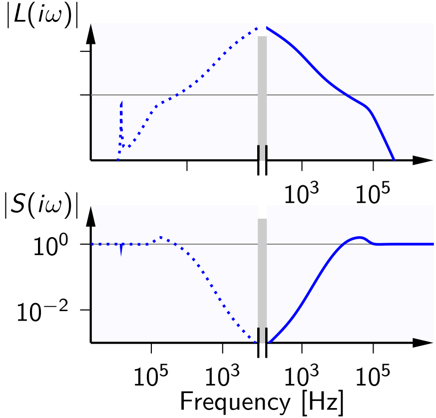


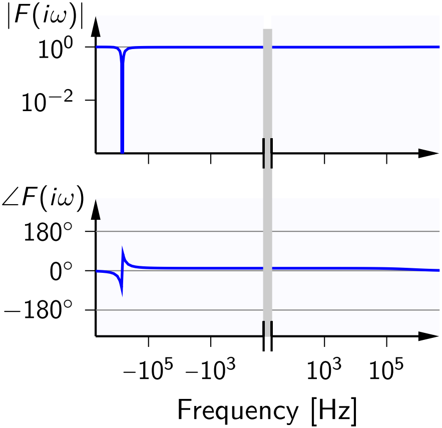
Control Strategies for Parasitic Modes (1/3)

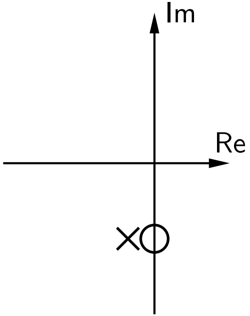
PI controller + 2nd order filter:





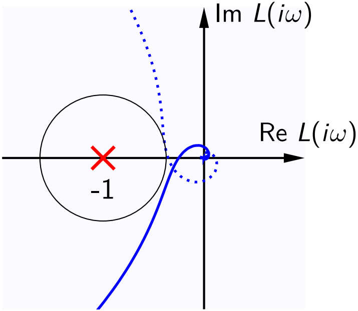
One-Sided Notch Filter

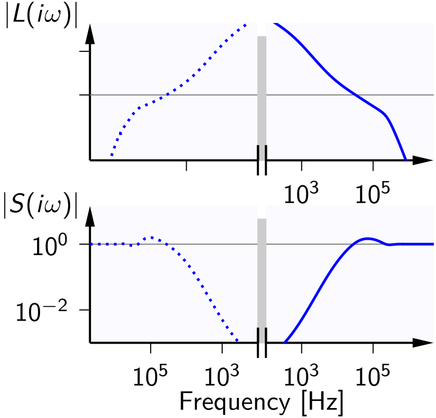




Control Strategies for Parasitic Modes (2/3)

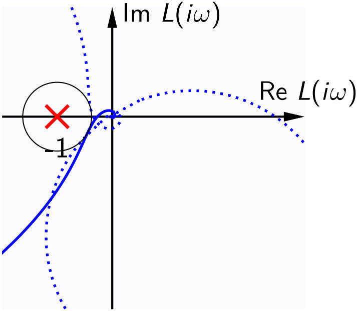
PI controller + one-sided notch filter + 2nd order filter:

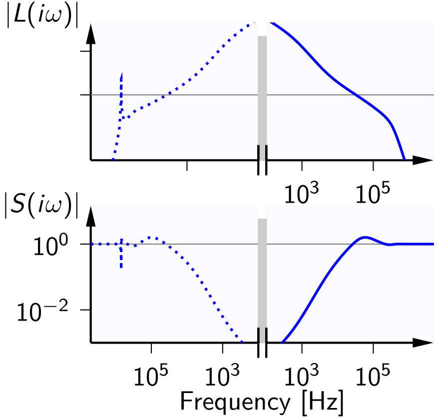




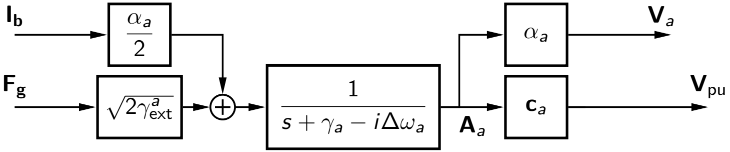
Control Strategies for Parasitic Modes (3/3)

PI controller + 3rd order filter,
adjusting phase of resonant "bulge":





Parasitic Modes



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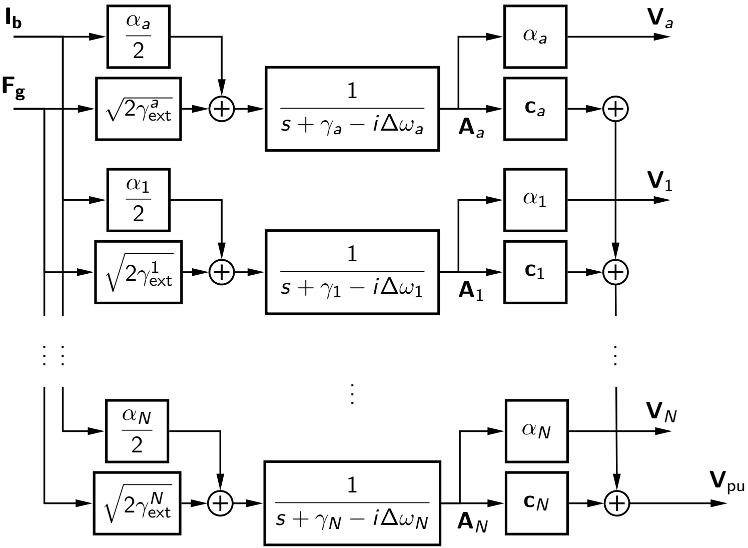

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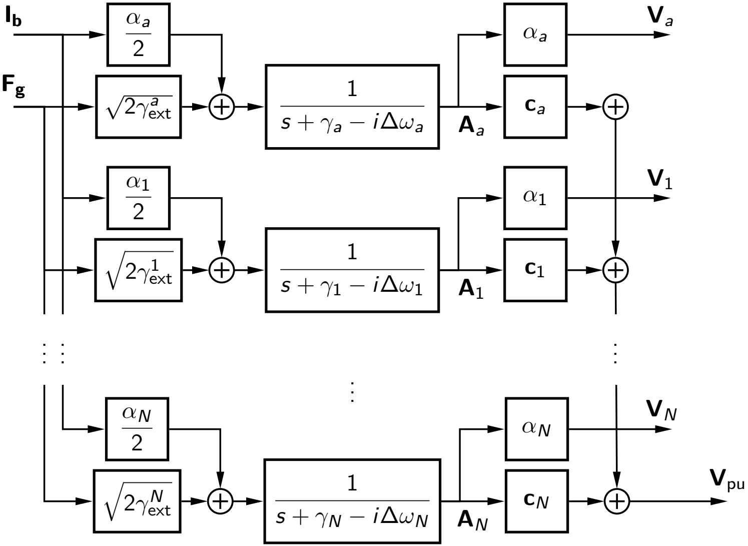


The important variable $\mathbf{V}_a$ is not measured!

Summary

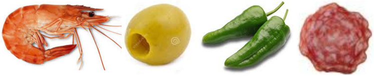
Considered some different aspects of the field control problem:

- Cavity modeling
- Complex-coefficient transfer functions
- Parasitic modes

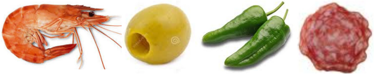
Very brief treatment, some more details can be found in:

- Complex-coefficient Systems in Control
  OT, B. Bernhardsson and C. Rivetta
  American Control Conference 2017, Seattle. (May 24–26, 2017)
- Cavity Field Control for High-Intensity Proton Accelerators
  OT
  Licentiate Thesis. (2017)

Thank you for listening!

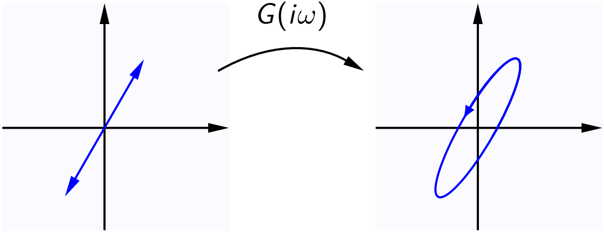


Extra slides



Effect of Signal with Specific Direction

$\boldsymbol{u}(t) = \boldsymbol{u}_0 \cdot \cos(\omega t)$



$$\boldsymbol{y}(t) = \boldsymbol{u}_0 \cdot \frac{1}{2}(G(i\omega)\mathrm{e}^{i\omega t} + G(-i\omega)\mathrm{e}^{-i\omega t})$$
$$= \boldsymbol{u}_0 \cdot \left[A_{\mathrm{Re}} \cos(\omega t + \phi_{\mathrm{Re}}) + iA_{\mathrm{Im}} \cos(\omega t + \phi_{\mathrm{Im}})\right]$$