Towards a better understanding of cavity field control

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Cavity Field Control

Some perspectives on the classic field control problem:
(coming from automatic control, high-intensity proton linac)
Cavity Field Control

Some perspectives on the classic field control problem:
(coming from automatic control, high-intensity proton linac)

- Cavity modeling, normalization
- Phasor diagrams, directionality
- Complex-coefficient systems
- Parasitic modes
\[
\frac{dV}{dt} = (-\omega_{1/2} + i\Delta\omega)V + RL\omega_{1/2}(2I_g + I_b)
\]

\[
P_g = \frac{1}{4} \frac{r}{Q} Q_{\text{ext}} |I_g|^2
\]
Accelerator Cavity Modeling (1/2)

\[ \frac{dA}{dt} = (-\gamma + i \Delta \omega)A + \sqrt{2} \gamma_{ext} F_g + \frac{\alpha}{2} I_b \]

- \( A \) – Mode amplitude \([\sqrt{J}]\)
- \( F_g \) – Forward wave \([\sqrt{W}]\)

\[ \frac{dV}{dt} = (-\omega_{1/2} + i \Delta \omega)V + R_L \omega_{1/2} (2 I_g + I_b) \]

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[Haus 1983]
\[
\frac{dA}{dt} = (-\gamma + i\Delta \omega)A + \sqrt{2}\gamma_{ext}F_g + \frac{\alpha}{2}I_b
\]

\(A\) – Mode amplitude \([\sqrt{J}]\)

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\(V = \alpha A \quad (\alpha = \sqrt{\omega_0(r/Q)})\)

\(P_g = |F_g|^2\)

\[
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\(P_g = \frac{1}{4} \frac{r}{Q} Q_{ext} |I_g|^2\)
Although we will soon normalize the model, the proposed parametrization [Waves & Field in Optoelectronics, Haus (1984)] has the following advantages:

- Cleaner expressions, e.g., $P_g = |F_g|^2$
- Mode states are not dependent on the particle velocity
- Cleaner treatment of parasitic modes of elliptical cavities
- Allows more direct derivation from Maxwell’s eqs.
Normalization

Normalization of accelerating mode dynamics around nominal operating point gives:

\[ Y(s) = \frac{\gamma}{s + \gamma - i\Delta\omega} \underbrace{P_{\text{cav}}(s)}_{U(s) + K_g D_g(s) + K_b D_b(s)} \]

- \( K_g, K_b \) – dimensionless constants, typically \( 1 \leq |K_g| \leq 2, 0 \leq |K_b| \leq 1 \)
\[
\frac{dA}{dt} = (-\gamma + i\Delta\omega)A + \sqrt{2}\gamma_{\text{ext}}I_g + \frac{\alpha}{2}I_b
\]

**Phasor Diagrams**

Mode amplitude \( A \) \([\sqrt{J}]\)

Terms of \( \frac{d}{dt}A \) \([\sqrt{J}/s]\)
Directionality in the Control Problem

\[ \frac{dA}{dt} = (-\gamma + i\Delta \omega)A + \sqrt{2\gamma_{\text{ext}}}l_g + \frac{\alpha}{2}l_b \]

Directionality of objective

Implications:
Actual performance not directly dependent on \( A \) and \( \phi \)

Optimal controller is not rotationally symmetric
Directionality in the Control Problem

\[
\frac{dA}{dt} = (-\gamma + i\Delta\omega)A + \sqrt{2}\gamma_{\text{ext}}l_g + \frac{\alpha}{2}l_b
\]

Directionality of objective

Directionality of disturbances

longitudinal focusing

acceleration

K\textsubscript{g}d\textsubscript{g}

\sqrt{2}\gamma_{\text{ext}}F_g

K\textsubscript{b}d\textsubscript{b}

\frac{\alpha}{2}l_b
Directionality in the Control Problem

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\frac{dA}{dt} = (-\gamma + i\Delta\omega)A + \sqrt{2\gamma_{ext}}l_g + \frac{\alpha}{2}l_b
\]

Implications:
- Actual performance not directly dependent on \( A \) and \( \phi \)
- Optimal controller is not rotationally symmetric
Control Theory for Complex-Coefficient Systems

\[
G(s) = \sum_{k=a,1,...} \frac{c_k \gamma_k}{s + \gamma_k - i\Delta\omega_k}
\]

- Standard control theory applies (if $T \mapsto \ast$ and $\int_0^\infty \mapsto \int_{-\infty}^\infty$), e.g., the Nyquist criterion; negative frequencies are significant.
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- MatLab handles complex-coefficient systems poorly
- Two useful applications: loop phase and parasitic modes
Loop Phase Adjustment

Open loop transfer function

\[ L(s) = P_{ca}(s)e^{-sL}e^{-i\theta} \cdot C_0(s)e^{i\theta_{adj}} = L_0(s)e^{i\delta} \]

Stability and robustness depends on loop phase adjustment error

\[ \delta = \theta_{adj} - \theta \]
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Stability and robustness depends on loop phase adjustment error
\[ \delta = \theta_{\text{adj}} - \theta \]

\[ \delta = 0^\circ \]
\[ \delta = \pi/6 \leftrightarrow 30^\circ \]
Loop Phase Adjustment

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Control Strategies for Parasitic Modes (1/3)

PI controller + 2nd order filter:
One-Sided Notch Filter

\[ |F(i\omega)| \]

\[ \angle F(i\omega) \]

Frequency [Hz]
Control Strategies for Parasitic Modes (2/3)

PI controller + one-sided notch filter + 2nd order filter:

\[ \text{Re } L(i\omega) \]
\[ \text{Im } L(i\omega) \]

\[ |L(i\omega)| \]
\[ |S(i\omega)| \]

Frequency [Hz]
PI controller + 3rd order filter, adjusting phase of resonant "bulge":
The important variable $V_a$ is not measured!
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Parasitic Modes

The important variable $V_a$ is not measured!
Summary

Considered some different aspects of the field control problem:

- Cavity modeling
- Complex-coefficient transfer functions
- Parasitic modes

Very brief treatment, some more details can be found in:

- Complex-coefficient Systems in Control
  OT, B. Bernhardsson and C. Rivetta

- Cavity Field Control for High-Intensity Proton Accelerators
  OT
Thank you for listening!
Extra slides
Effect of Signal with Specific Direction

\[ u(t) = u_0 \cdot \cos(\omega t) \]

\[ y(t) = u_0 \cdot \frac{1}{2} (G(i\omega)e^{i\omega t} + G(-i\omega)e^{-i\omega t}) \]

\[ = u_0 \cdot \left[ A_{\text{Re}} \cos(\omega t + \phi_{\text{Re}}) + iA_{\text{Im}} \cos(\omega t + \phi_{\text{Im}}) \right] \]