Determination and compensation of the “reference surface” from redundant sets of surface measurements

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We want to characterize the departure of optical surfaces to ideal geometry (flat, sphere, ellipse) with accuracy $< 1$ nm.

No one is able to produce such a surface.

- Manufacturing problems
- Stability on the long term (thermal, supporting)

Measurement are always made with respect to a Reference.

Reference can be:

- A particular surface: interferometry
- A calibration result: (LTP)

Invariance properties can be used to generate an estimate of the reference:

- Flat surface, sphere and cylinder

Self referencing can be generalized to any shape (with help of computers):

- Using the self-consistency of a measurement set
- Akin to stitching problem
Modeling the reference problem

- **Interferometric measurements**
  - Measures the wavefront difference between the surface under test (SUT) and a reference (Ref)
  - Extra unknowns: tilts (T) & mean distance (d)
  - \( M = S - R + (T + d) \)
  - Other instrumental issues:
    - Linearity and uniformity of the phase shifts
    - Distortion of the imaging system

- **Slope measurements (LTP)**
  - Measures directly the local slope to a linear constant \( k \)
  - \( k \) must be experimentally determined
  - unknown: the linearity correction (and Tilt)
  - \( M = k S - L(M) + T \)
  - Origin of the correction:
    - Variation of the optical path with local slopes
How redundancy may help

- **Object and Reference functions are unique**
- Any measurement is a point to point linear combination of the object and reference

- By taking measurements at different position the point to point relationship is broken and shared with a larger set of points

- The data from such sets of points should be consistent with the uniqueness of the Object and Reference + unique distances and tilts for each frame

- This allows for determining extra experimental parameters, tilts, distances and recovering the two functions

- The problem is solved globally avoiding propagations of errors which distort the reconstruction

- Consistency is never perfect, noise, incomplete description of the acquisition

- Solution need to be found in a maximum likelihood sense
Analogy with ptychography

- Ptychography is a technique of CDI (computed diffraction imaging) where a light probe of limited area is scanned on a sample, and far field diffraction recorded.

- Each subfield is reconstructed with CDI reconstruction algorithm.

- Constraints coming from the overlapping area and the constant probe function are used to refine iteratively the unknown phases.

- As an outcome, both object and probe functions are reconstructed.

- Difference with reference problem:
  Ptychography is non-linear (multiplicative). Reference problem is linear (additive), so in principle simpler.

Thibault et al., *Science*, 321 (5887): 379-382
Application to LTP

- **The LTP calibration curve**: \( \text{slope} = f(\text{spot position}) \)
  - is not a straight line
    - Non linearity deviation \( \pm 2 \mu\text{rad} \text{ typ.} \) on the full 8 mrad stroke
    - Many evidences from Round Robins and periodic controls of ref. artifacts

- **Causes**
  - The slight position change of the return pencil beam and
  - Imperfect optical elements: lens, prisms, mirrors
  - Presence of local defects (in thick glass elements)

- **Stability**
  - Calibration curves are stable on short term (measurement of 1 piece)
  - Variable on long term with configurations

- **Redundant measurement allow altogether**
  - To characterize the deviation from linearity
  - Recover the surface profile
  - Extend the angular range of measurement by stitching
Method

- Record a dense set of slope profiles
  - tilting the SUT incrementally between 2 profiles
  - In order to cover most of the rectangle defined by the length of the SUT and the LTP slope range

- Establish the equations relating the measured slope values \( M(x,p) \) to the unknowns
  - \( S(x) \) real SUT Slope profile
  - \( C(m) \) linearity Correction of the LTP
  - \( T(p) \) tilt angle of the optics table
  \[
  M(x,p) = S(x) - C(M(x,p)) + T(p)
  \]

- Discretize the equations for computer solving
  - Measurements can be taken on a discrete set of position points \( x_i \)
  - \( C(m) \) needs to be interpolated between the slope points \( m_j \) of a discretization grid

*If the tilt rotation axis is not on the SUT the x positions should be corrected and interpolated*
Interpolation

- Interpolation should be local to preserve a "point" to "point" relationship
  - Eg. polynomial approximation on a small number of neighboring points

\[ t \in [t_i, t_{i+1}] \Rightarrow f(t) = \sum_{i-k}^{i+k} F_n P_n(t-t_i) \]

- Then, assuming that the rotation axis is on the surface \(^1\) the variables can be written in vector form as

\[ [S] = \begin{bmatrix} S(x_1) \\ S(x_n) \end{bmatrix}, \quad [C] = \begin{bmatrix} C(m_1) \\ C(m_q) \end{bmatrix}, \quad [T] = \begin{bmatrix} T_1 \\ T_p \end{bmatrix} \]

and the set of equations as

\[ [Q] [S] [C] [T] = [M_1] [M_p] \]

Where \([Q]\) is large sparse matrix and \([M]\) the vector of all measured points

- This set of equations is overdefined \(iI\) can be only solved in a maximum likelihood sense

1. If not, a local interpolation should be also applied in x to care for irregular sampling positions
M43 mirror (R=43.3m) was measured independently with ESRF and SOLEIL LTPs (2009)

Because of the short radius a stitching procedure is used.

When global redundancy based stitching is applied a close agreement is found (red and blue curves)

Discrepancies are found when conventional end to end stitching is used
The linearity correction should be a constant of the optical system.

Small variations may come from different X positions of the SUT on the LTP bench, since the return optical path is slightly different.

A reasonable day to day consistency is observed.
- HZB, ESRF, Elettra measurements of the Zeiss sphere
- Redundant stitching procedure applied to SOLEIL data

It should be noted that the NOM autocollimator (HZB) from Elcomat receives a precise linearity compensation.
**Method**

Record several 16 x 12 mm frames
tiled with 2 – 3 mm steps (Dx)

The measured heights are related to the unknowns

- $S(x,y)$ real height map of the SUT
- $R(x,y)$ real height map of the Reference
- $T(n)$ spurious displacement vector of the translation table at each step (Z and tilt angles)

by

$$M(x+n \text{Dx}, y)=S(x, y) - R(x, y) + (T_0(n) + T_1(n) x + T_2(n) y)$$

No special discretization step needed if the step Dx is an integer number of pixels
otherwise a local polynomial interpolation might be applied

Generate and solve the corresponding system of equations

$$\begin{bmatrix} Q \\ S \\ R \\ T \end{bmatrix} = \begin{bmatrix} M_1 \\ \vdots \\ Mp \end{bmatrix}$$
Solving the equation system

- **The matrix Q is a huge but very sparse matrix**
  - eg recording a 150 mm X 12 strip in 3 mm (144 px) steps requires recording 44 frames, amounting to 20 M data points = number of equations
  - The reconstructed image is 6970 x 580 = 4 M points
  - The reference is 780 x 580 = 0.3 M points, + 130 tilt-displacements
  - But Q has only 5 non null element per row

- **The equation matrix is solved iteratively under Matlab**
  - It requires ~ 10-15 minutes on the computer cluster of SOLEIL

- **Convergence requires some caution**
  - Namely, starting from reasonable estimates of the reference and tilts
  - Reference is estimated by averaging all the frames together and is subtracted from the measurements. The computed Ref is actually a correction.
  - This way of estimating the reference implies that reference and SUT are reconstructed to a unknown uniform curvature (namely the average SUT radius)
Stitching measurement of a Ø200 zerodur flat

Global stitching

Propagative stitching

Differential interferometry of the full surface

R. Mercier, Lab. Charles Fabry, IOGS (1996)
Profiles from stitching interferometry and LTP

Raw stitched height profiles compared to heights from integrated LTP data

Stitched height profiles after addition of a convex 83 km curvature
Ion beam polished mirror on a 2 mm track

1 collaboration SESO, SOLEIL, LMA, ISP system, under ANR grant 09-NANO-008-AXOC
The principle of using a set of redundant measurements to recover both, the measured object, and a reference or correction function, has been proved effective.

It can be applied to stitching measurement problems
- on slope (LTP)
- on surface heights (interferometry)

Work is still needed to
- improve the convergence
- condition the equation matrices (a key point not yet studied)
- refine the frame spacing to avoid periodic artifacts