Self calibration of the accuracy in the measurement of a mirror bender

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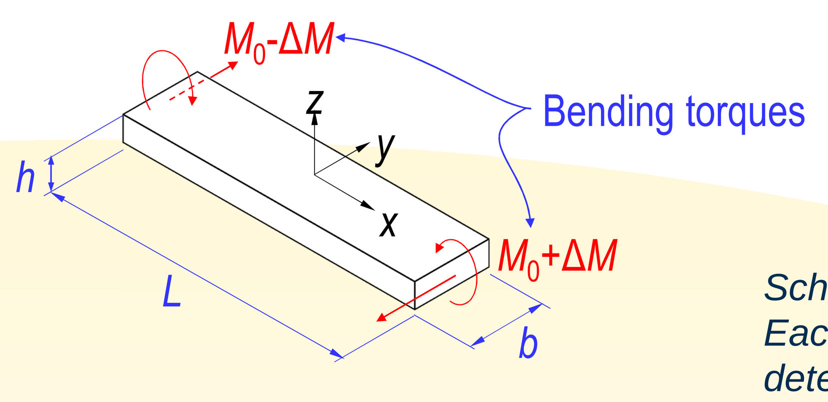
Abstract

During the calibration of a mirror bender, either with one or two independent bending actuators, the mirror surface is measured several times under different bending conditions. By doing this, one obtains multiple and redundant measurements of the surface under test. For all the measurements, the polishing error remains invariant, while the elastic deformation of the substrate and the instrument error are changing. The deformation of the substrate is well described by the elastic beam theory, and for each configuration of the bender, it can be completely determined by only two parameters. We propose in this work a method to combine the redundant measurements of the surface to reconstruct the bending parameters and the polishing slope error, as well as the instrument error of the LTP. The method consists in building a system of equations with the different contributions to the measurement. The system is linearized by assuming that both the slope error and the instrument error can be written in a basis of cubic splines. The error-free slope function can then be reconstructed from the solution.

Theory



Scheme of a mirror bender. Each value of M_0 and ΔM determine a different bending condition for the bender

The elastic deformation of the mirror is accurately described by the Euler-Bernouilli equation:

$$E\frac{bh^3}{12}\frac{d^2z(x)}{dx^2} = M_0 + \Delta M\frac{2x}{L}$$

The solution is a cubic polynomial, threfore the slope of the deformation is a quadratic polynomial

Profile of the deformation $z_R(x) = \frac{U}{2}x^2 + \frac{V}{3}x^3$ $\qquad U = \frac{12M_0}{Eh^3b_0} = \frac{1}{R}$

Slope of the deformation $\frac{dz_R(x)}{dx} = Ux + Vx^2$ $\qquad V = \frac{12\Delta M}{Eh^3b_0L} = \frac{E_3}{3}$

The measured slope has contributions from three sources

- Polishing slope error, which does not depend on the bending condition.
- Bending figure, which has a pure polynomial form that depends only on the bending condition.
- The instrument error, which depends only on the measured angle.

$$M(x,k) = S(x) + T_k + U_kx + V_kx^2 + L(M(x,k))$$

measurement at point x, in the k^{th} bending condition — *Slope error at point x* — *Deformation at point x in the k^{th} bending condition* — *Linearity error of the measurement*

Such system can be linearized by expressin *S* and *L* in a base of spline functions

$$M(x,k) = \sum_{i=1}^{I} S_iB_i(x) + T_k + U_kx + V_kx^2 + \sum_{j=1}^{J} L_jB_j(M(x,k))$$

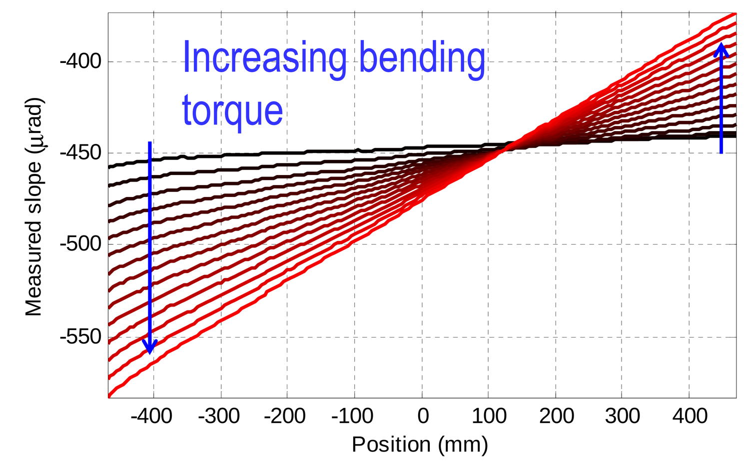
By taking several measurements (*k*=1,…,N) at different bending conditions, one can build the following linear system, whose solution provides the slope error, the bending parameters, and the instrument error.

$$\begin{bmatrix} M(x_1,1) \\ \vdots \\ M(x_X,1) \\ \hline \vdots \\ \hline M(x_1,N) \\ \vdots \\ M(x_X,N) \end{bmatrix} = \begin{bmatrix} B_1(x_1) & \dots & B_I(x_1) & B_1(M(x_1,1)) & \dots & B_J(M(x_1,1)) & 1 & 0 & 0 & x_1 & 0 & 0 & x_1^2 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_1(x_X) & \dots & B_I(x_X) & B_1(M(x_X,1)) & \dots & B_J(M(x_X,1)) & 1 & 0 & 0 & x_X & 0 & 0 & x_X^2 & 0 & 0 \\ \hline & & & & & & & \vdots \\ \hline B_1(x_1) & \dots & B_I(x_1) & B_1(M(x_1,N)) & \dots & B_J(M(x_1,N)) & 0 & 0 & 1 & 1 & 0 & x_1 & 0 & 0 & x_1^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_1(x_X) & \dots & B_I(x_X) & B_1(M(x_X,N)) & \dots & B_J(M(x_X,N)) & 1 & 0 & 1 & 1 & 0 & x_X & 0 & 0 & x_X^2 \end{bmatrix} \begin{bmatrix} S_1 \\ \vdots \\ S_I \\ \hline L_1 \\ \vdots \\ L_J \\ \hline T_1 \\ \vdots \\ T_K \\ \hline U_1 \\ \vdots \\ U_K \\ \hline V_1 \\ \vdots \\ V_K \end{bmatrix}$$

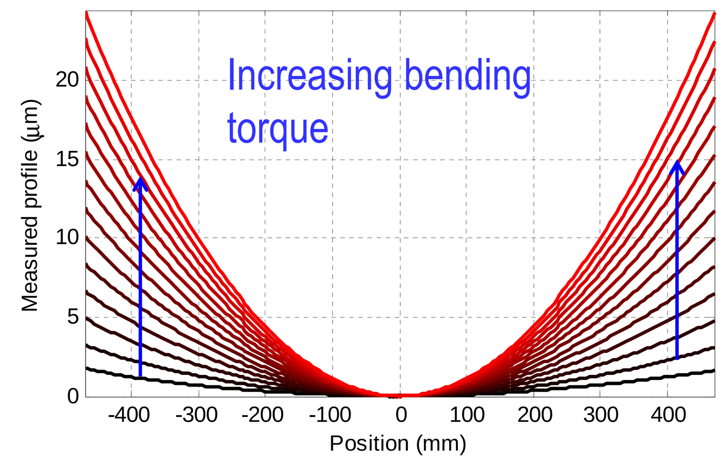
Results

The proposed method has been used to characterize a single-bender actuator for the CLAESS beeamline at ALBA. A total of 40 LTP traces at different bending conditions have been combined.

Raw data

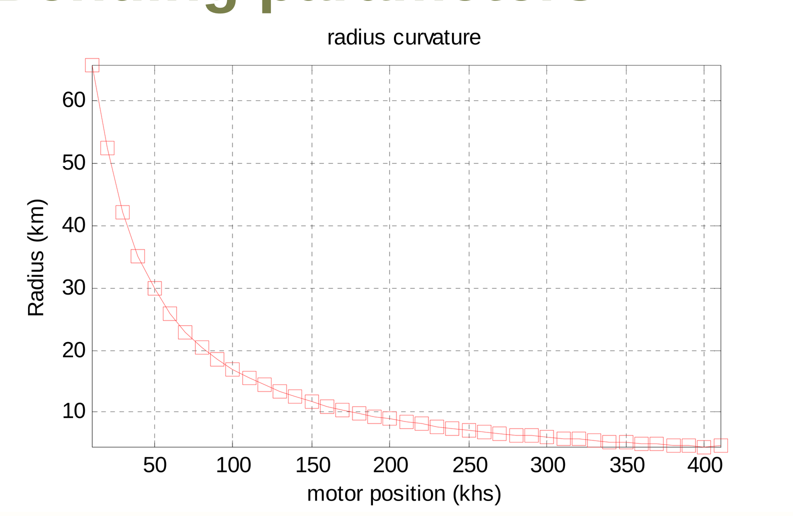


Collection of slope measurmenets obtained during the calibration of a 900 mm mirror bender.

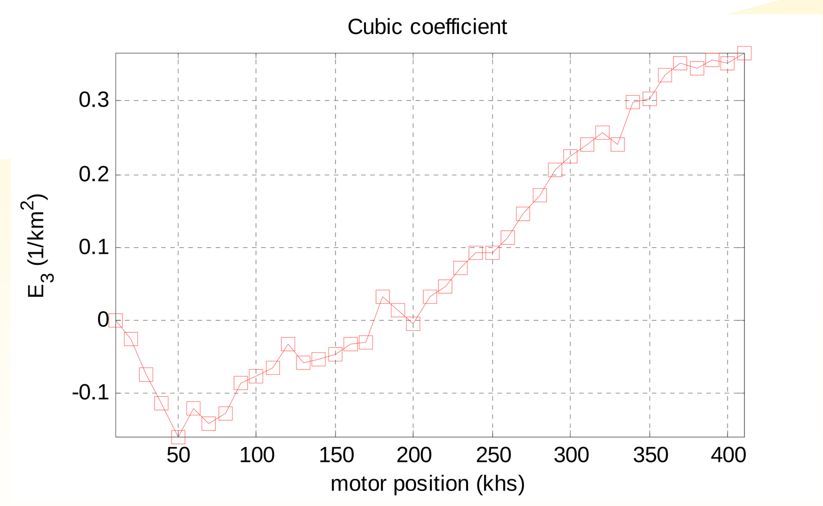


Collection of profilesmeasured during the calibration of a 900 mm mirror bender.

Bending parameters

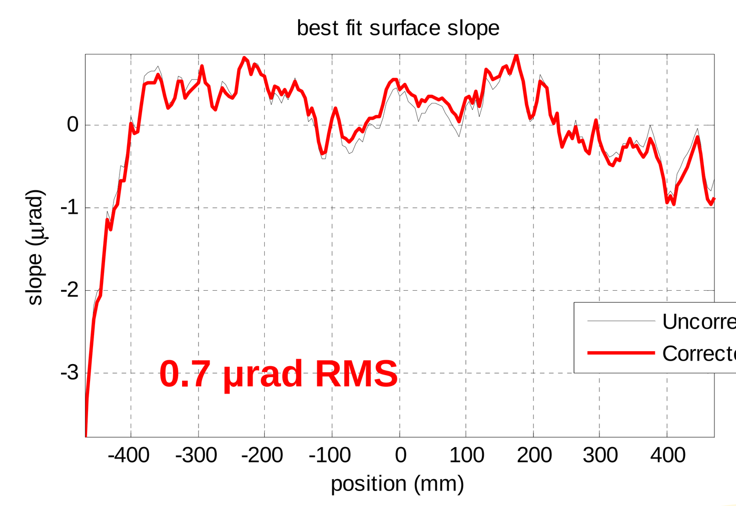


Measured radius of curvature as a function of the average bending torque. It follows the $1/M_0$ law.

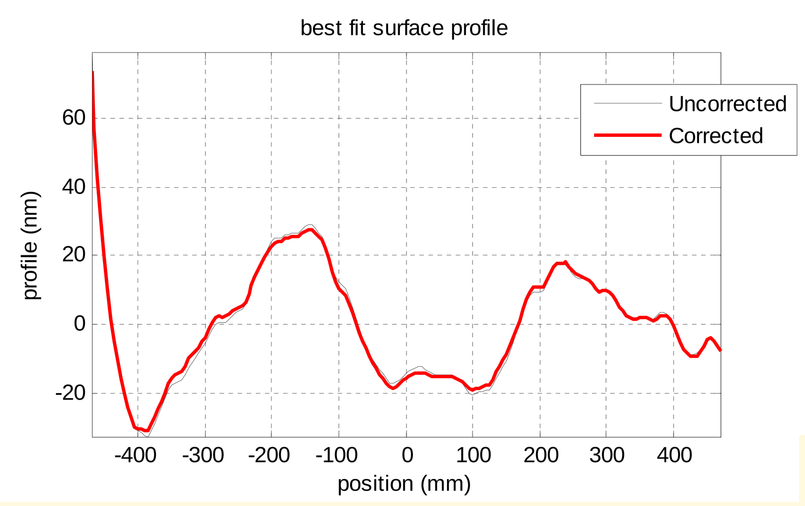


Measured cubic term of the ellipse. It reveals a slight asymetry between the two end-torques.

Slope error

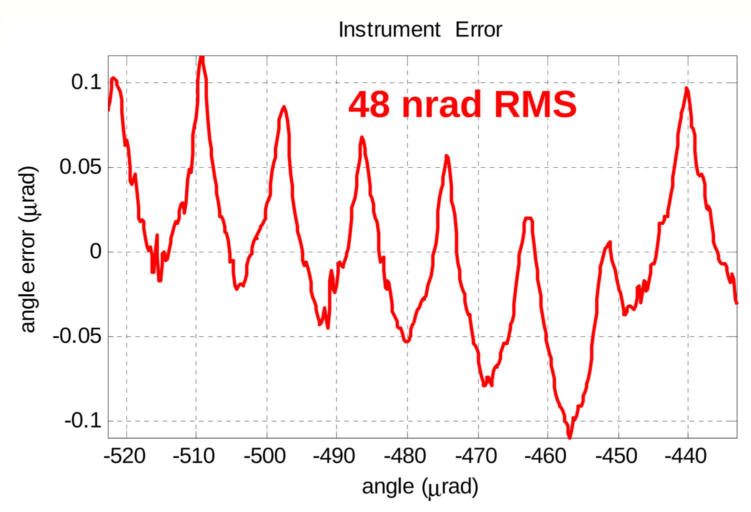


Residual slope error obtained from the reconstruction, compared with one directly measured. The small difference is due to instrument error.

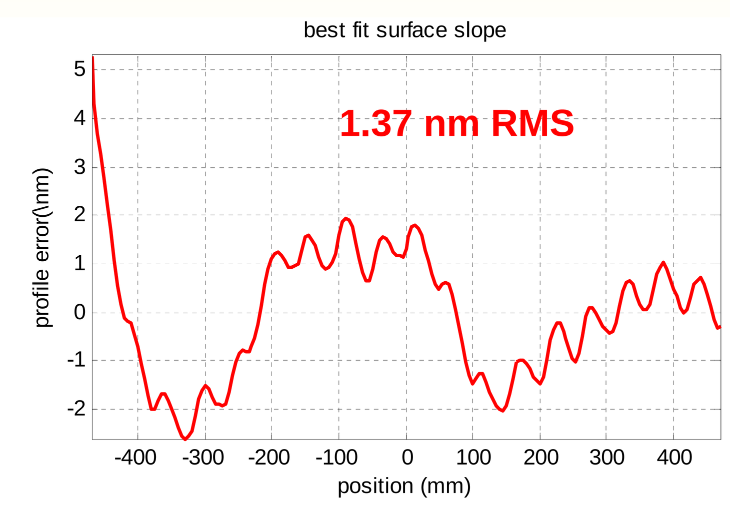


Residual profile obtained from the reconstruction, compared with one integrated from a direct measured.

Instrument error



Instrument error in the range of the measurement. It shows the usual periodicity of the autocollimator.



Error of a single measurent with respect to the reconstruction

References

[1] F. Polack, M. Thomasset, S. Brochet, A. Rommeveaux, *Nucl. Instrum. Meth. Phys. Res. A* **616** (2010) 207