Metrology of trapezoidal shape mirror benders

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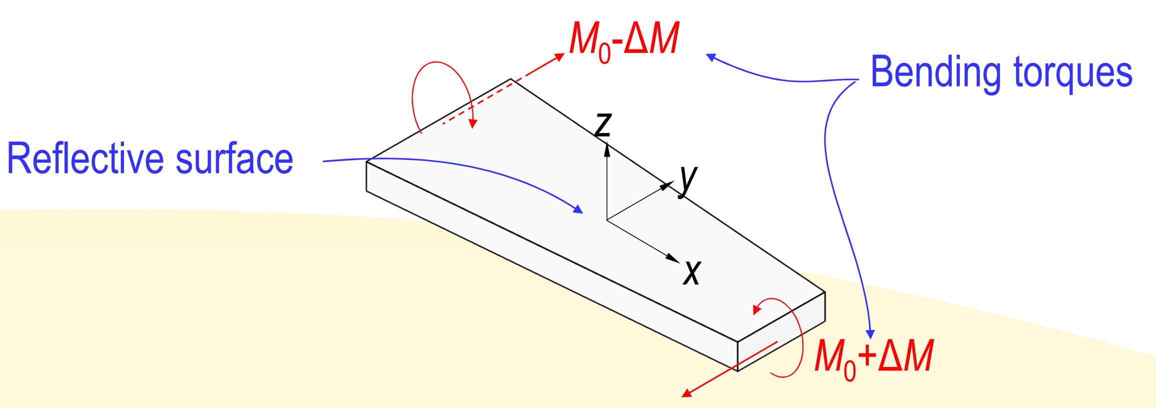
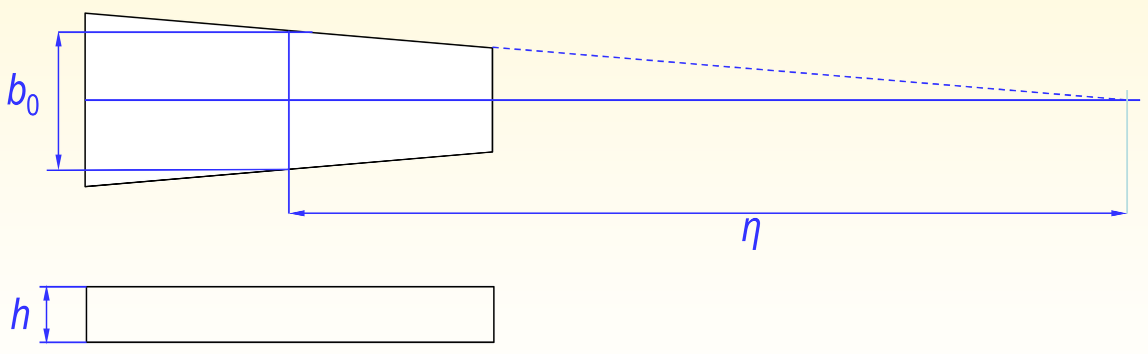
Abstract

When a high demagnification focusing is required for a plano-elliptical mirror, the required ellipse cannot be approximated by the 3rd order polynomial that can be reached by introducing two bending moments onto a solid, elastic parallelepiped. A typical approach to improve this is to cut the mirror bulk into a trapezoidal shape, in a way that the required ellipse can be approximated also to higher polynomial orders. In this case, the introduced deformation is described by an infinite series, which depends on three parameters: the two bending moments, and the width variation ratio. In this case, the optimization of the mirror cannot be done by simply fitting the measured profile to a 3rd degree polynomial, and identifying the ellipse parameters from the polynomial coefficients, since this leads to biased results. In this work we propose a method to accurately fit the ellipse parameters in terms of the elastic deformation of the beam and the residual slope error. This permits to fit the ellipse parameters accurately, independently of the polishing slope error.

Elastic deformation of a trapezoid





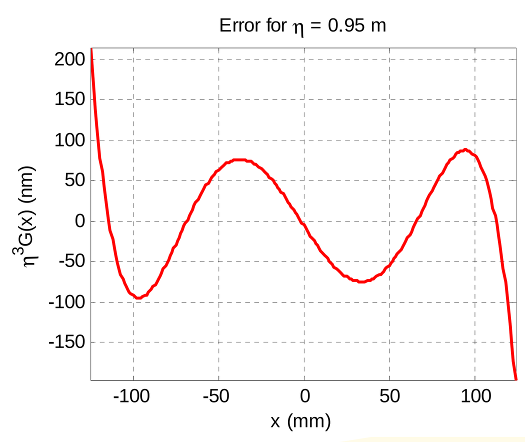
The width of the mirror is given by a linear function: $$b(x) = b_0\left(1+\frac{x}{\eta}\right)$$

The elastic deformation of the mirror is ruled by the Euler-Bernouilli equation:

$$E\,\frac{h^3 b_0}{12}\left(1+\frac{x}{\eta}\right)\frac{d^2 z(x)}{dx^2} = M_0 + \Delta M\,\frac{2x}{L}$$

The solution is typically written as a 4th order polynomial plus an error term

$$z_T(x) = T_2x^2 + T_3x^3 + T_4x^4 + T_3\eta^3 G\left(\frac{x}{\eta}\right)$$



The error term is small, and often can be neglected. Then, one fits the measured profile to a 4th order polynomial, to determine the error.

Alternatively we propose the following expression of the solution:

$$z_T(x) = T_2x^2 + T_3\left[3\eta x^2 - 6\left(\eta^3 + \eta^2 x\right)\log\left(1+\frac{x}{\eta}\right)\right]$$

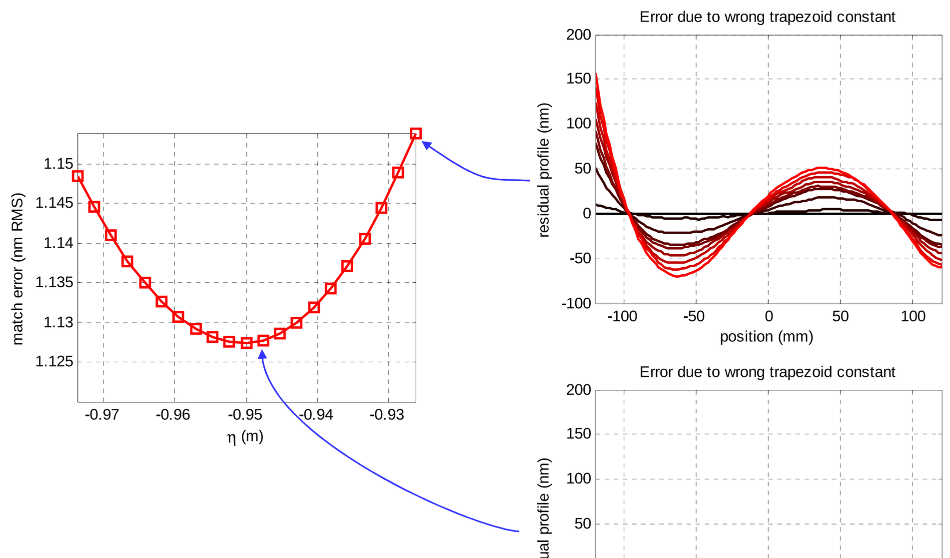
- It allows to decouple the elastic deformation from the polishing error.
- It provides self-consistent ellipse coefficients, allowing exact optimization of the mirror figure.
- It is a linear combination of functions that do not depend on the bending torques, therefore it allows a simple fitting by the LSM method.
- All the coefficients to be fit, are independent each other.
- Allows to predict the slope error at the nominal ellipse.

$$T_2 = \frac{6M_0}{Eh^3b_0} \quad \Leftrightarrow \quad E_2 = \left(\frac{1}{p}+\frac{1}{q}\right)\frac{\cos\alpha}{4}$$

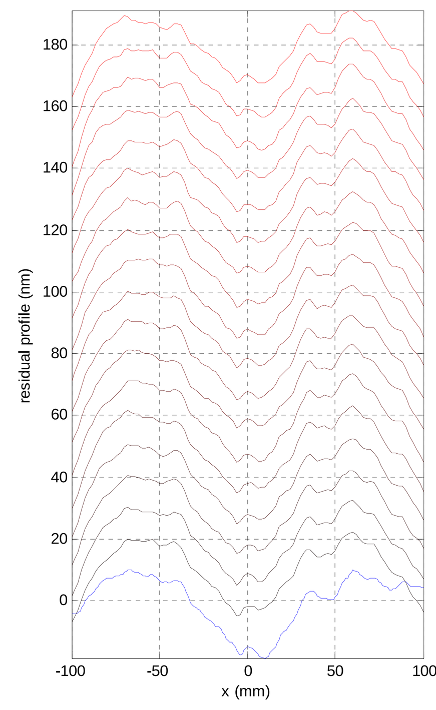
$$T_3 = \frac{4\Delta M}{Eh^3b_0L} - \frac{2M_0}{Eh^3\eta b_0} \quad \Leftrightarrow \quad E_3 = \left(\frac{1}{p^2}-\frac{1}{q^2}\right)\frac{\sin 2\alpha}{16}$$

Slope error decoupling

The width ratio of the mirror 1/η is normally a source of error. It can be fit by minimizing the differences between residuals for different bending conditions. Once this is done, the residual slope error is independent of the bending condition.



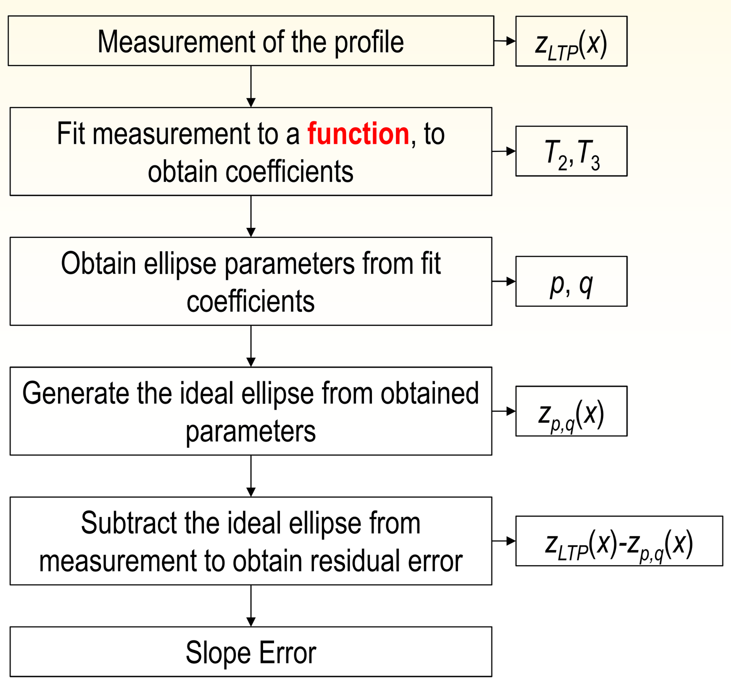
Differences of slope error for different bending conditions

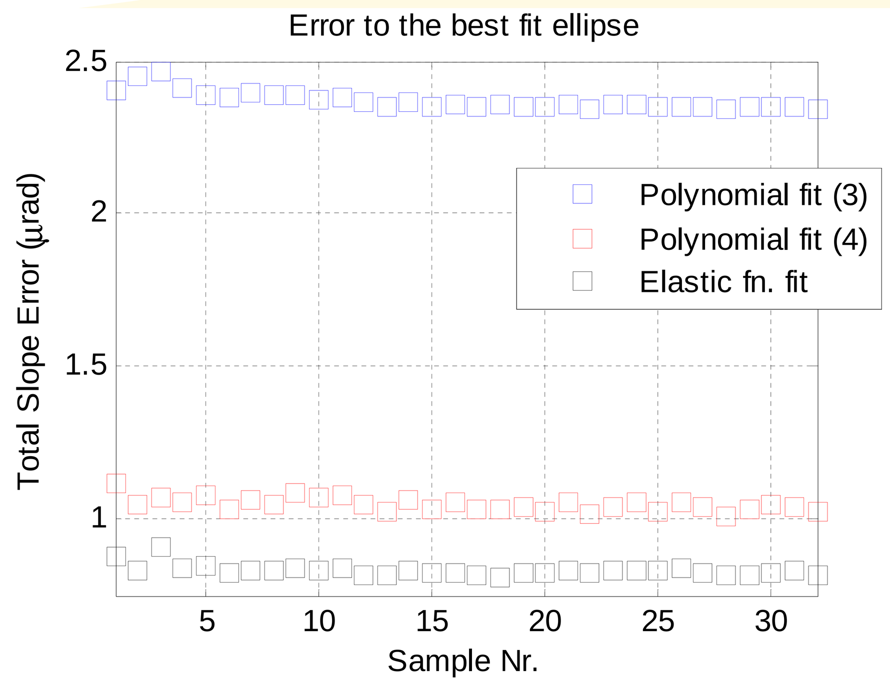


Residual profile for different bending conditions. Blue corresponds to mirror in a kinematic mount

Ellipse parameters

Since the mirror deformation is described with high accuracy, the estimation of the deformation parameters is accurate, which allows to fit the right ellipse and minimize the error.





Slope error with respect to the best fit ellipse, for different bending conditions, and using different deformation functions

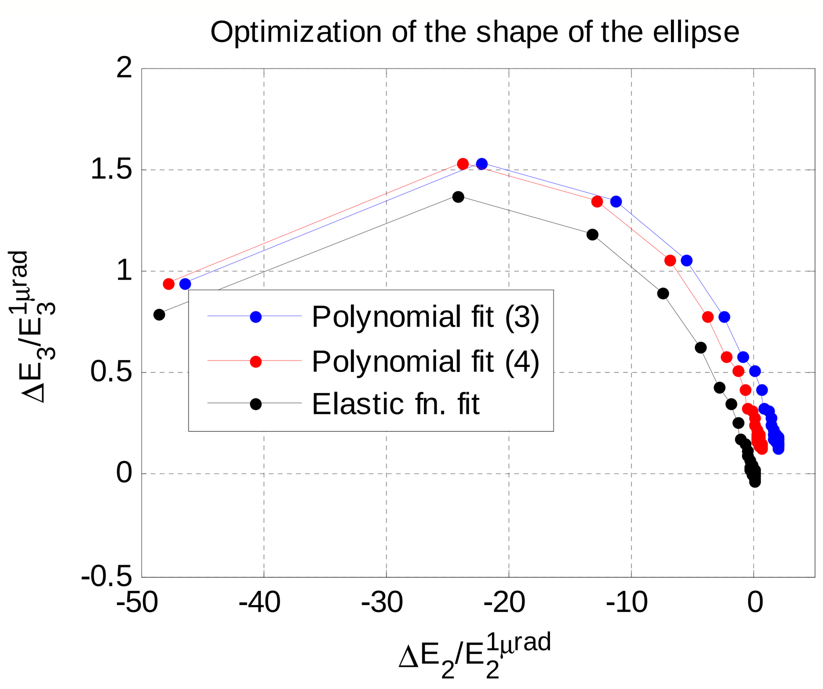
If the measured profile is fit to a polynomial, the resulting ellipse parameters are not accurately determined, what contributes to the residual slope error.

$$\zeta_E^2 = \frac{\Delta E_2^2}{3}L^2 + \frac{\Delta E_3^2}{20}L^4$$

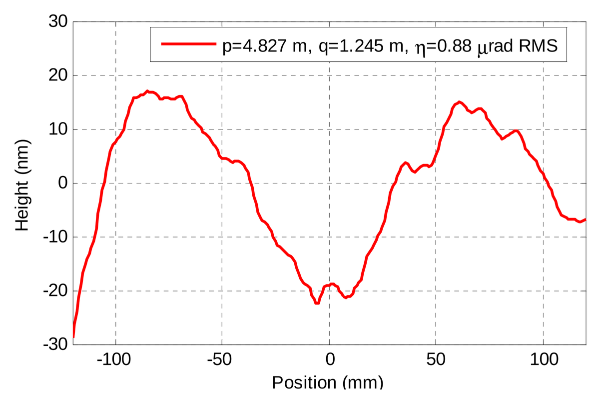
Nominal ellipse fitting

By using the proposed fitting, one can accurately adjust the bending torques so as to obtain the required nominal ellipse, still preserving the slope error measured at the mirror being flat.

The propose fitting is used in an automated procedure to adjust the mirror bending torques.



Path of the bender in the E_2,E_3 configuration space during automatic optimization. To use polynomial fit would lead to a biased ellipse.



p=4.825 m q=1.250 m

Residual error with respect to a best fit ellipse obtained by simplex optimization of p,q. The resulting ellipse is virtually identical to the nominal one.