Metrology of trapezoidal shape mirror benders
Josep Nicolas
CELLS-ALBA, 08290 Cerdanyola del Vallès, Spain

Abstract

When a demagnification focusing is required for a plano-elliptical mirror, the required ellipse cannot be approximated by the 3rd order polynomial that can be reached by introducing two bending moments onto a solid, elastic parallelepiped. A typical approach to improve this is to cut the mirror bulk into a trapezoidal shape, in a way that the required ellipse can be approximated also to higher polynomial orders. In this work we propose a method to accurately fit the ellipse parameters in terms of the elastic deformation of the beam and the residual slope error. This permits to fit the ellipse parameters accurately, independently of the polishing slope error.

Elastic deformation of a trapezoid

The width of the mirror is given by a linear function:

\[ b(x) = b_0 \left(1 + \frac{x}{\eta}\right) \]

The elastic deformation of the mirror is ruled by the Euler-Bernouilli equation:

\[ E I \frac{d^4 z(x)}{dx^4} = M_{x} + \Delta M_{x} \frac{2x}{L} \]

The solution is typically written as a 4th order polynomial plus an error term

\[ z(x) = T_1 x^2 + T_3 x^4 + T_5 x^6 + T_7 x^8 \left[ x \left(1 + \frac{x}{\eta}\right) \right] \]

The error term is small, and often can be neglected. Then, one fits the measured profile to a 4th order polynomial, to determine the error.

Alternatively we propose the following expression of the solution:

\[ z(x) = T_1 x^2 + T_3 x^4 + 3T_5 x^6 + 6T_7 x^8 \left[ x \left(1 + \frac{x}{\eta}\right) \right] \]

- It allows to decouple the elastic deformation from the polishing error.
- It provides self-consistent ellipse coefficients, allowing exact optimization of the mirror figure.
- It is a linear combination of functions that do not depend on the bending torques, therefore it allows a simple fitting by the LSM method.
- All the coefficients to be fit, are independent each other.
- Allows to predict the slope error at the nominal ellipse.

\[ T_1 = \frac{6M_{x}}{EIb_0} \quad E_x = \frac{1}{p} \left(1 - \frac{1}{q}\right) \cos \alpha \quad E_y = \frac{1}{4} \]

\[ T_3 = \frac{4AM_{x}}{EIb_0} \quad E_x = \frac{1}{p} \left(1 - \frac{1}{q}\right) \sin 2\alpha \quad E_y = \frac{1}{16} \]

Slope error decoupling

The width ratio of the mirror 1/\eta is normally a source of error. It can be fit by minimizing the differences between residuals for different bending conditions. Once this is done, the residual slope error is independent of the bending condition.

Ellipse parameters

Since the mirror deformation is described with high accuracy, the estimation of the deformation parameters is accurate, which allows to fit the right ellipse and minimize the error.

If the measured profile is fit to a polynomial, the resulting ellipse parameters are not accurately determined, what contributes to the residual slope error.

\[ \eta^2 \frac{x^2}{L^2} \frac{\Delta z}{\Delta x} = \frac{M_{x}^2}{3} L^2 + \frac{M_{x}^2}{20} L^2 \]

Nominal ellipse fitting

By using the proposed fitting, one can accurately adjust the bending torques so as to obtain the required nominal ellipse, still preserving the slope error measured at the mirror being flat.

The propose fitting is used in an automated procedure to adjust the mirror bending torques.

Path of the bender in the E_x E_y configuration space during automatic optimization. To use polynomial fit would lead to a biased ellipse.