Magnetic Measurements and Faraday's law;

have we Understood the Paradoxes?

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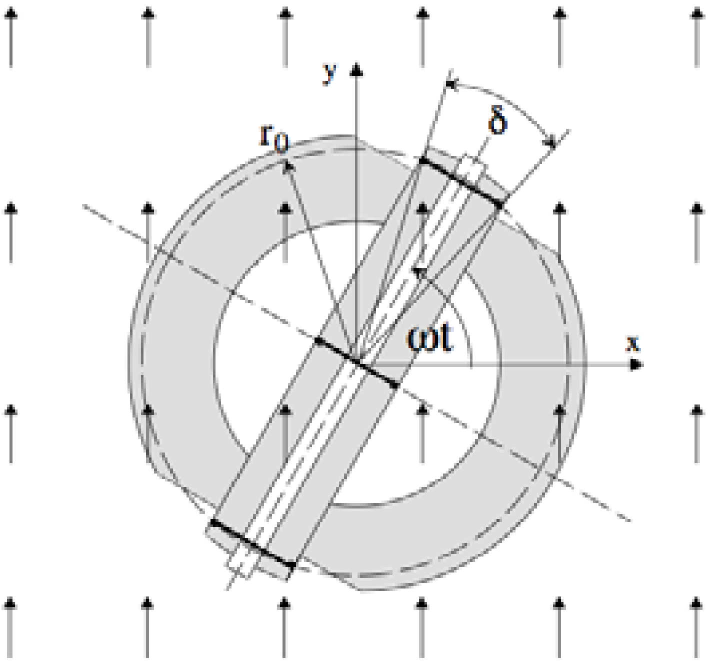


$$U(\partial\mathscr{A}) = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathscr{A}} \mathbf{B}\cdot\mathbf{da}.$$

The “flux rule”

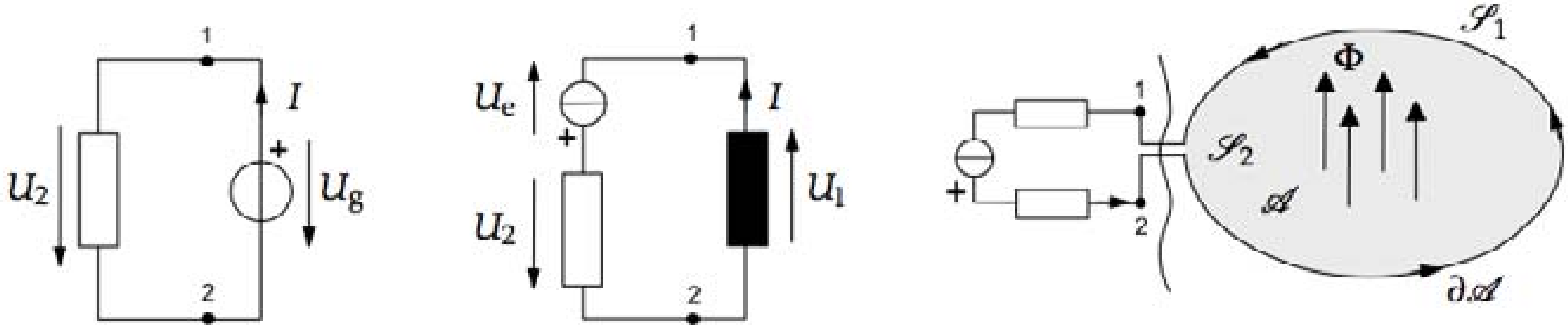
$$\Phi(t) = N\ell \int_{\varphi-\delta/2}^{\varphi+\delta/2} B_r(r_0,\varphi) r_0 \mathrm{d}\varphi$$
$$= \sum_{n=1}^{\infty} S_n^{\tan}\left[B_n(r_0)\sin(n\omega t + n\Theta) + A_n(r_0)\cos(n\omega t + n\Theta)\right],$$

where

$$S_n^{\tan} := \frac{2N\ell r_0}{n}\sin\left(\frac{n\delta}{2}\right)$$

Circuit Equations: Load and Generator Convention

$$U_g = U_2 = U_{12} = -\frac{\mathrm{d}}{\mathrm{d}t}\Phi(\mathscr{A}) .$$

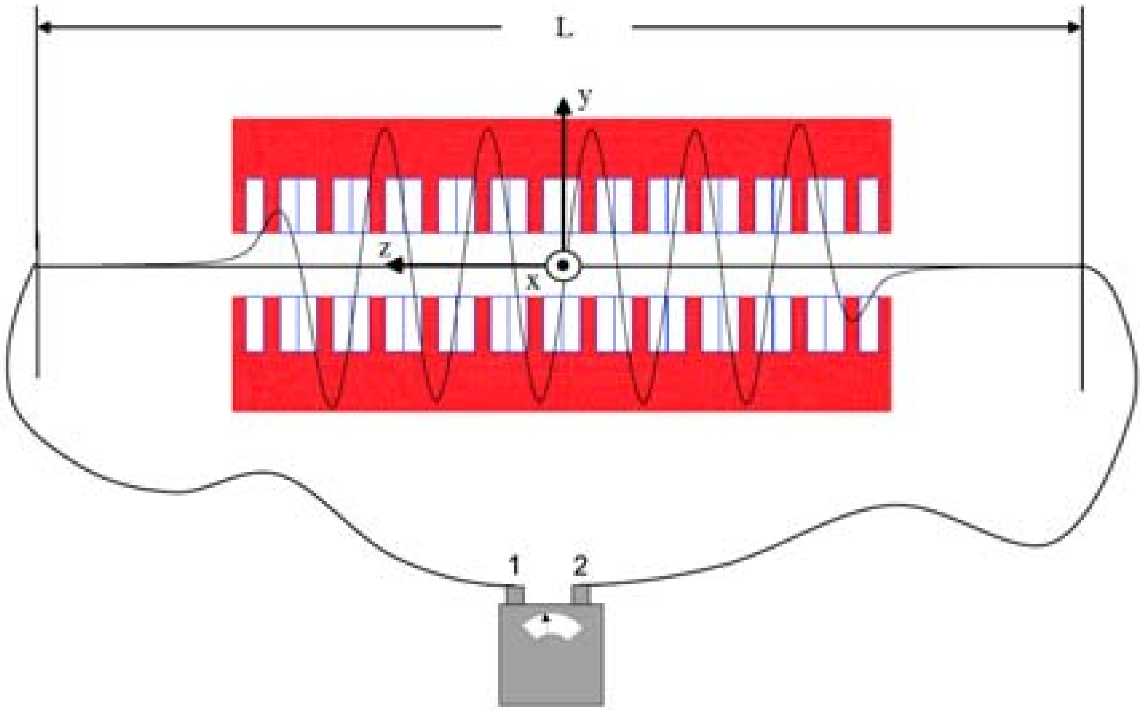


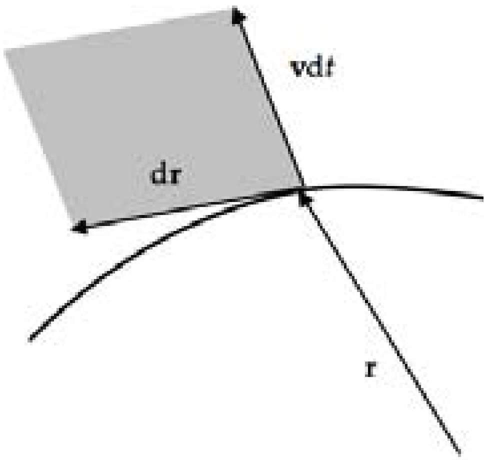
$$U_1 = U_e - U_2 = -U_{12} = \frac{\mathrm{d}}{\mathrm{d}t}\Phi(\mathscr{A}) .$$

$$U = -N\frac{\mathrm{d}}{\mathrm{d}t}\Phi(\mathscr{A}) - R\,I ,$$

Stretch Wire Measurements (Motional EMF)

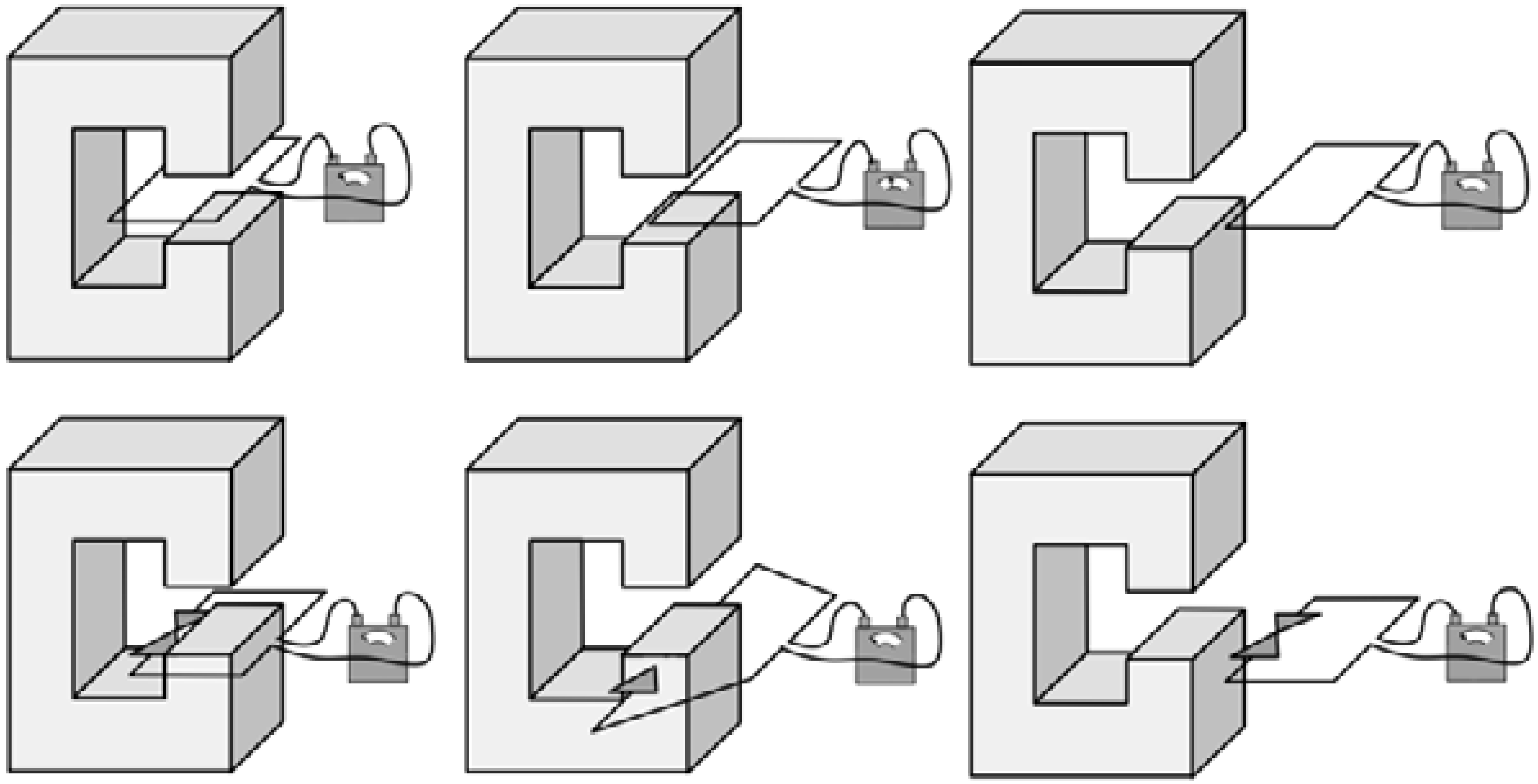
$$F = Q\,\mathbf{v} \times \mathbf{B},$$





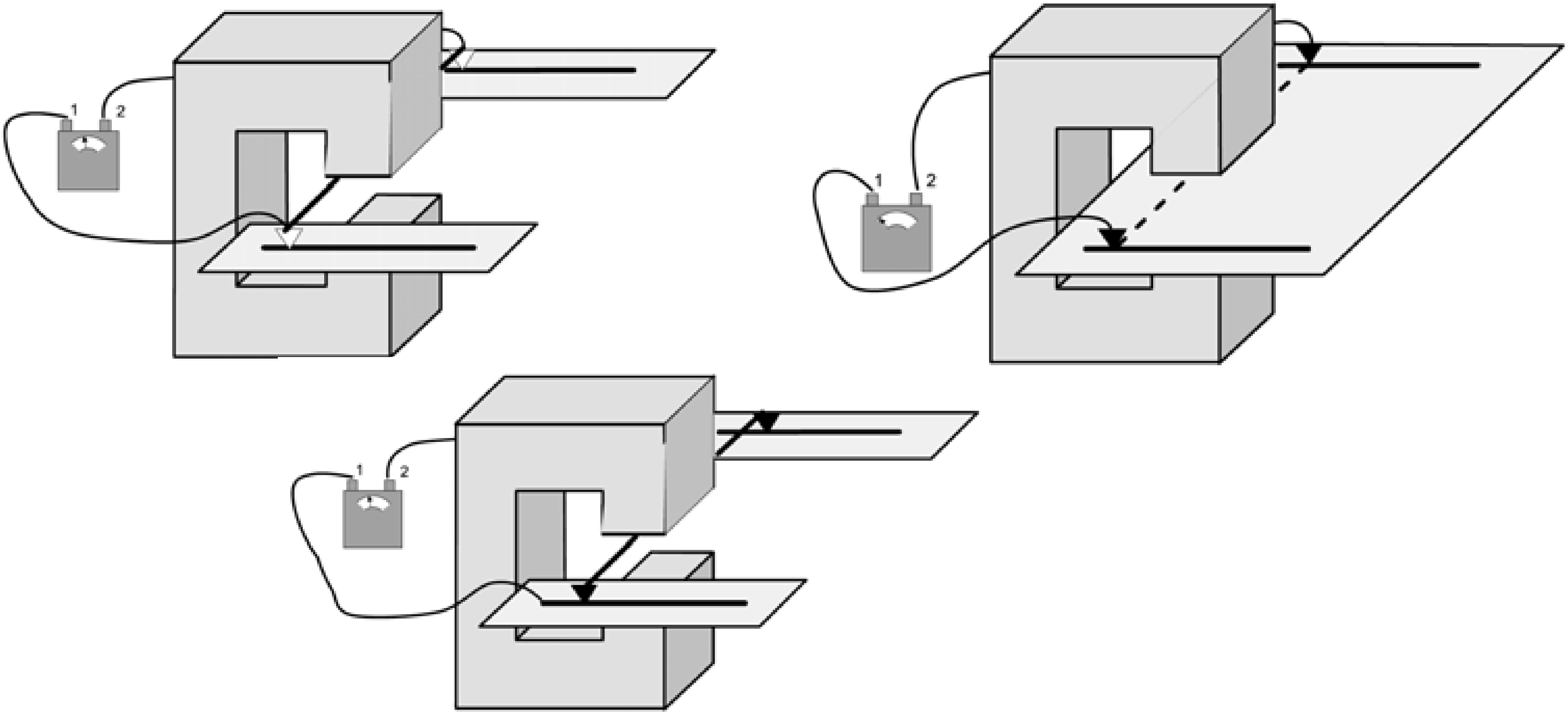
$$U = \int_{\mathscr{S}} (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{r} = -\int_{\mathscr{S}} \mathbf{B} \cdot (\mathbf{v} \times \mathrm{d}\mathbf{r}) = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathscr{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$$

Experiment by Karl Hering (1908)

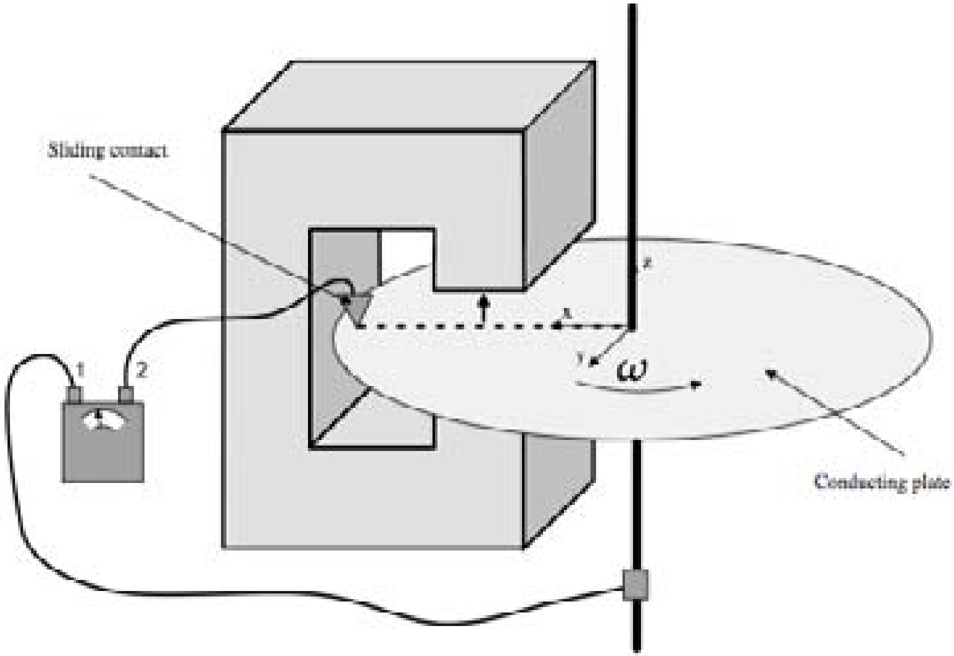


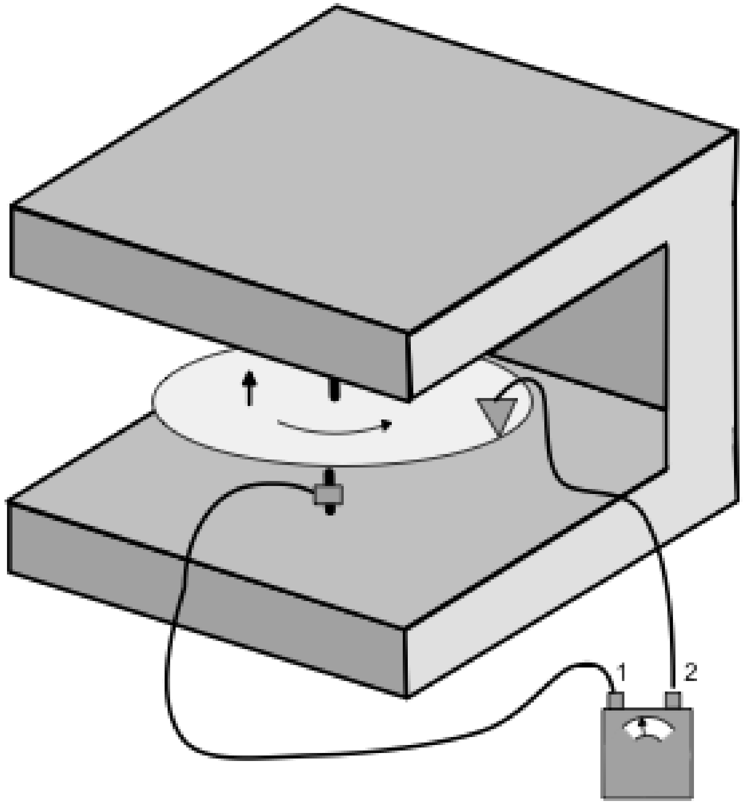
Technical Relevance for Measurements

- Are slip rings a problem?
- Is there a difference between these three setups (Stretch wire, Feynman plate)?



The Homopolar Generator; a Paradox?





282 000 Entries for the Paradox

faraday paradox

Page 5 of about 282,000 results (0.14 seconds)

[Latest News and Information on **Faraday Paradox**]
www.blogieboo.com/Affiliate-Marketing/.../**Faraday-Paradox**.html - Cached
31 Aug 2011 – is if the field is rotating with the magnet or not. I learned this is the **Faraday Paradox**. en.wikipedia.org I would have to read the article ...

YES_O_ERHS : Message: **faraday paradox**
groups.yahoo.com/group/YES_O_ERHS/message/983?var=1 - Cached
2 posts - 1 author - Last post: 17 Oct 2006
The **Faraday paradox** (or **Faraday's paradox**) is an experiment that illustrates the limitations of Michael Faraday's law of electromagnetic **...**

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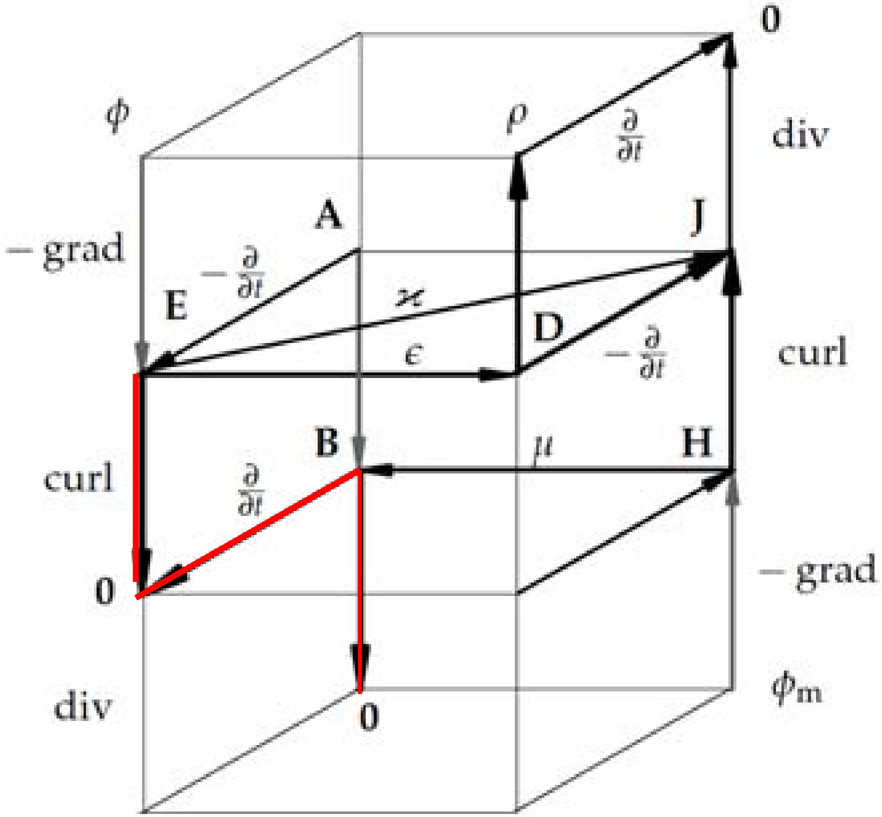
Faraday Paradox - Google Books
books.google.com › Science › Physics
The **Faraday paradox** is an experiment that illustrates Michael Faraday's law of electromagnetic induction. Faraday deduced this law in 1831, after inventing **...**

[NPA Chat] vXB and the **Faraday Paradox**
worldnpa.org/pipermail/memberschat_worldnpa.../006306.html - Cached
16 Sep 2009 – When it exists in conjunction with EM induction it can be merged into **Faraday's** law. It is the motionally dependent aspect of **Faraday's** law. ...

What is the Situation in the Literature

- ➔ Those who present the right approach, don’t care about the applications
  - Always use the fundamental laws
- ➔ Those who present the application often tweak the Maxwell equations to the problem at hand.
  - The reasoning for that often includes confusing arguments
    - The loop must keep its material identity
    - The loop must rest in the moving media
    - Faraday’s law is not valid in such and such cases
    - The local form is more general
    - For Hering: The flux was never linked to the coil



$$\operatorname{curl}\mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D},$$

$$\operatorname{curl}\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B},$$

$$\operatorname{div}\,\mathbf{B} = 0,$$

$$\operatorname{div}\,\mathbf{D} = \rho.$$

The simple form of constitutive equations are only true for linear (field-indep.), homogeneous (position-independ), isotropic (direction-indep.), lossless, and stationary media

Required: Orientable manifolds, orientation, frame, metric, continuity, contractible domains

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Required: Orientable manifolds

orientation, frame, metric, continuity

No switches, no Moebius strips

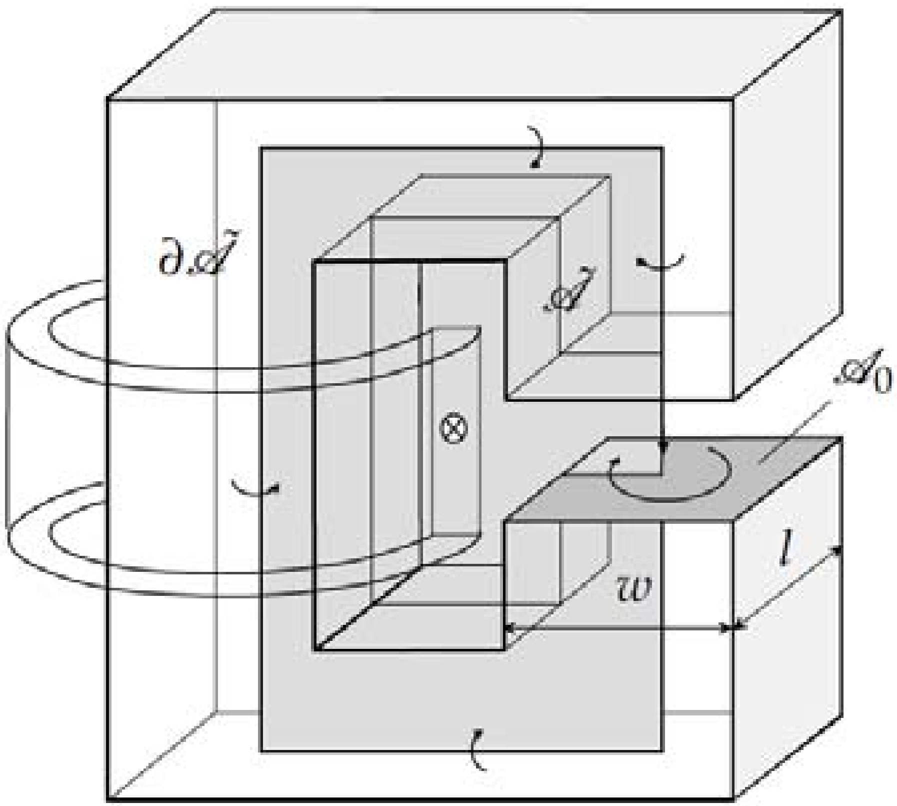
No switches, no Moebius strips, no internal boundaries, no holes in surfaces, no bubbles in volumes

internal boundaries

Constitutive Equations I

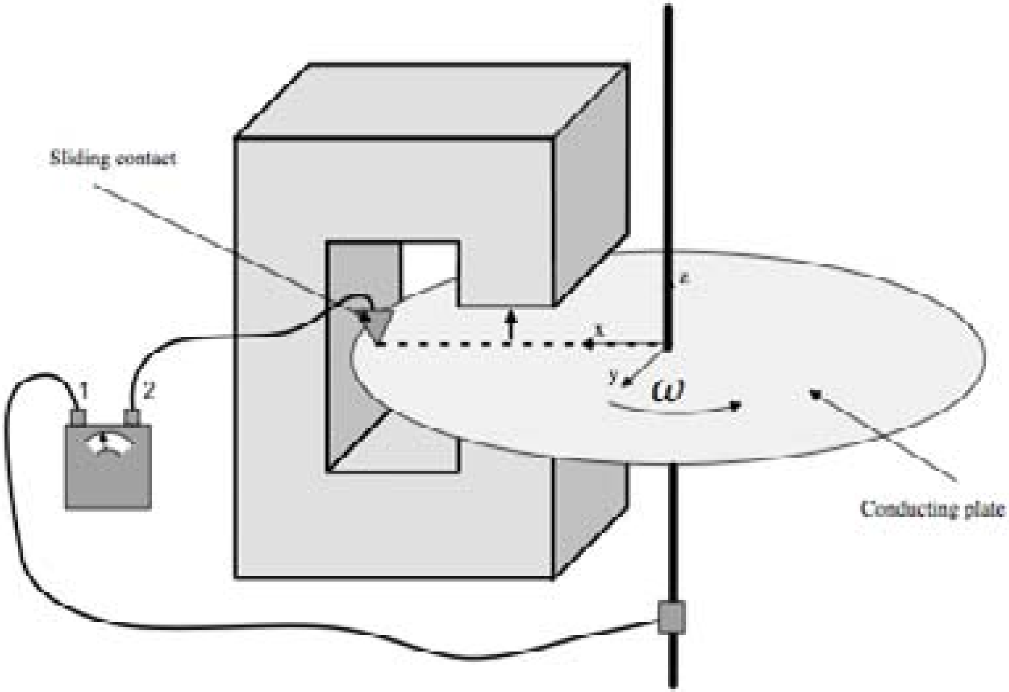
$$\int_{\partial \mathscr{A}} \mathbf{H} \cdot d\mathbf{r} = \int_{\partial \mathscr{A}} \mathbf{H} \cdot \mathbf{t}\, ds = \int_{\mathscr{A}} \mathbf{J} \cdot \mathbf{n}\, da$$

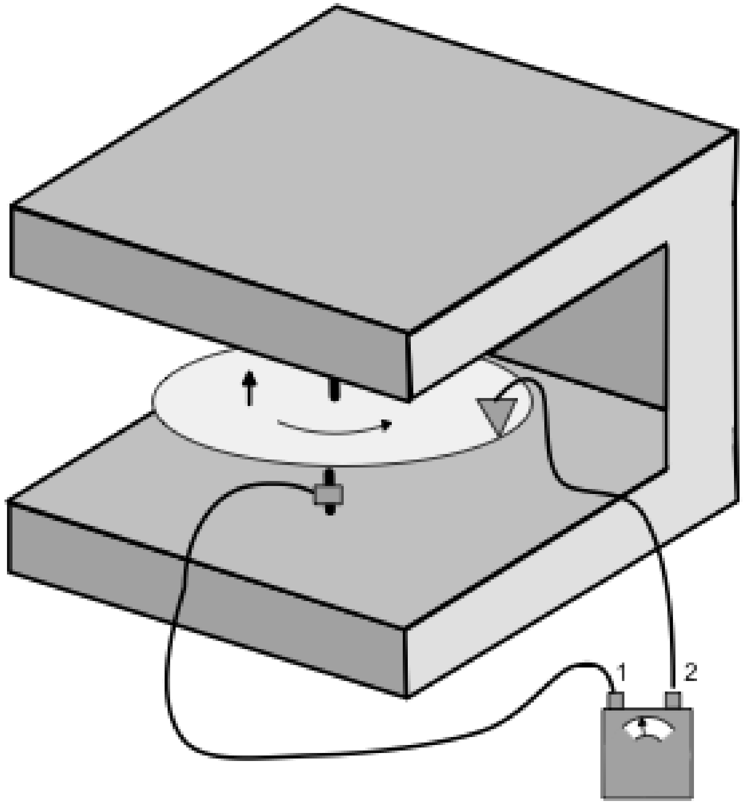
$$H_{\text{iron}}\, s_{\text{iron}} + H_0\, s_0 = N\, I$$



$$\Phi = \int_{\mathscr{A}_0} \mathbf{B} \cdot d\mathbf{a}$$

$$\frac{B_i}{\mu_i} = \frac{\Phi}{a_i\, \mu_i}$$





Einstein: All physics is local

$$
\begin{aligned}
\mathbf{E}' &= \mathbf{E} + \mathbf{v} \times \mathbf{B}\,, \\
\mathbf{H}' &= \mathbf{E} - \mathbf{v} \times \mathbf{D}\,, \\
\mathbf{M}' &= \mathbf{M} + \mathbf{v} \times \mathbf{P}_{\mathrm{el}}\,, \\
\mathbf{J}' &= \mathbf{J} - \mathbf{v}\rho\,, \\
\mathbf{B}' &= \mathbf{B}\,, \\
\mathbf{D}' &= \mathbf{D}\,, \\
\mathbf{P}'_{\mathrm{el}} &= \mathbf{P}_{\mathrm{el}}\,, \\
\rho' &= \rho\,,
\end{aligned}
$$

$$\mathbf{J}' = \varkappa \mathbf{E}',$$

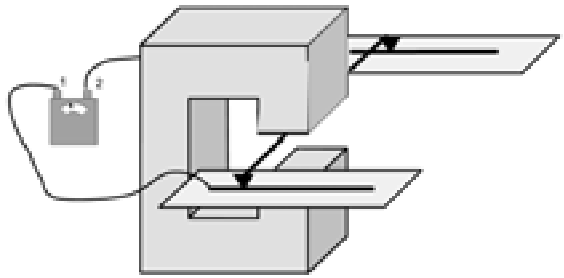
$$\mathbf{J} - \mathbf{v}\rho = \varkappa(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,.$$

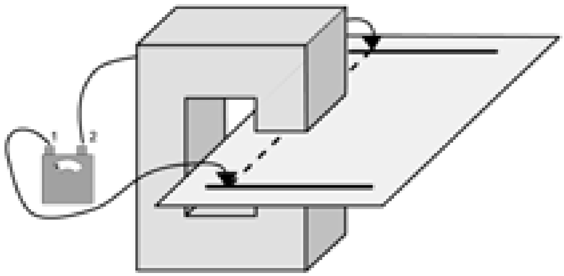
$$\operatorname{curl} \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B},$$

Feynman: When the material in the circuit is changing, we must return to the basic laws. The correct physics is always given by the two basic laws.

The Network Equations (Steady State)

$$\int_{\partial\mathcal{A}} \mathbf{E}\cdot \mathrm{d}\mathbf{r} = 0 = U_{12} + \int_{\mathcal{S}_1} \frac{1}{\varkappa a} I \,\mathrm{d}s + \int_{\mathcal{S}_2} \frac{1}{\varkappa a} I \,\mathrm{d}s + \int_{\mathcal{S}_2} (\mathbf{v}_{\mathrm{m}} \times \mathbf{B})\cdot \mathrm{d}\mathbf{r}$$



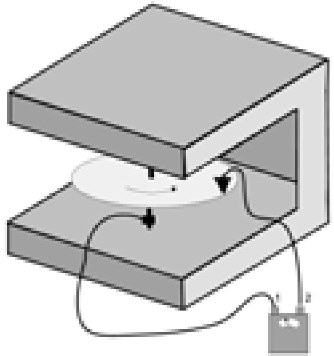


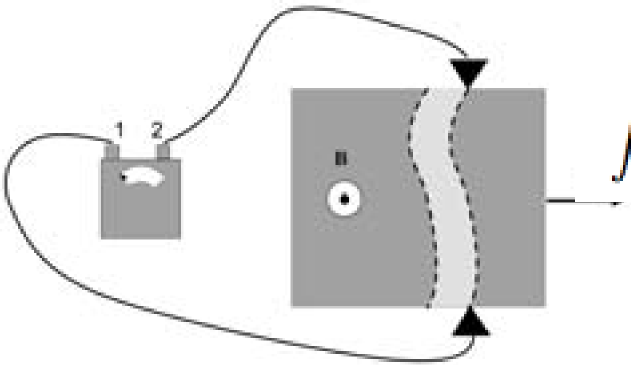
$$\int_{\partial\mathcal{A}} \mathbf{E}\cdot \mathrm{d}\mathbf{r} = 0 = U_{12} + \int_{\mathcal{S}_1} \frac{1}{\varkappa a} I \,\mathrm{d}s + \int_{\mathcal{S}_2} \frac{1}{\varkappa a} I \,\mathrm{d}s$$

$$U_{12} = -R\, I,$$

$$\int_{\partial\mathcal{A}} \mathbf{E}\cdot \mathrm{d}\mathbf{r} = 0 = U_{12} + \int_{\mathcal{S}_1} \frac{1}{\varkappa a} I \,\mathrm{d}s + \int_{\mathcal{S}_2} \frac{1}{\varkappa a} I \,\mathrm{d}s + \int_{\mathcal{S}_2} (\mathbf{v}_{\mathrm{m}} \times \mathbf{B})\cdot \mathrm{d}\mathbf{r}$$

$$U_{12} + R\, I = \frac{1}{2}\omega\, r_0^2\, B_0$$



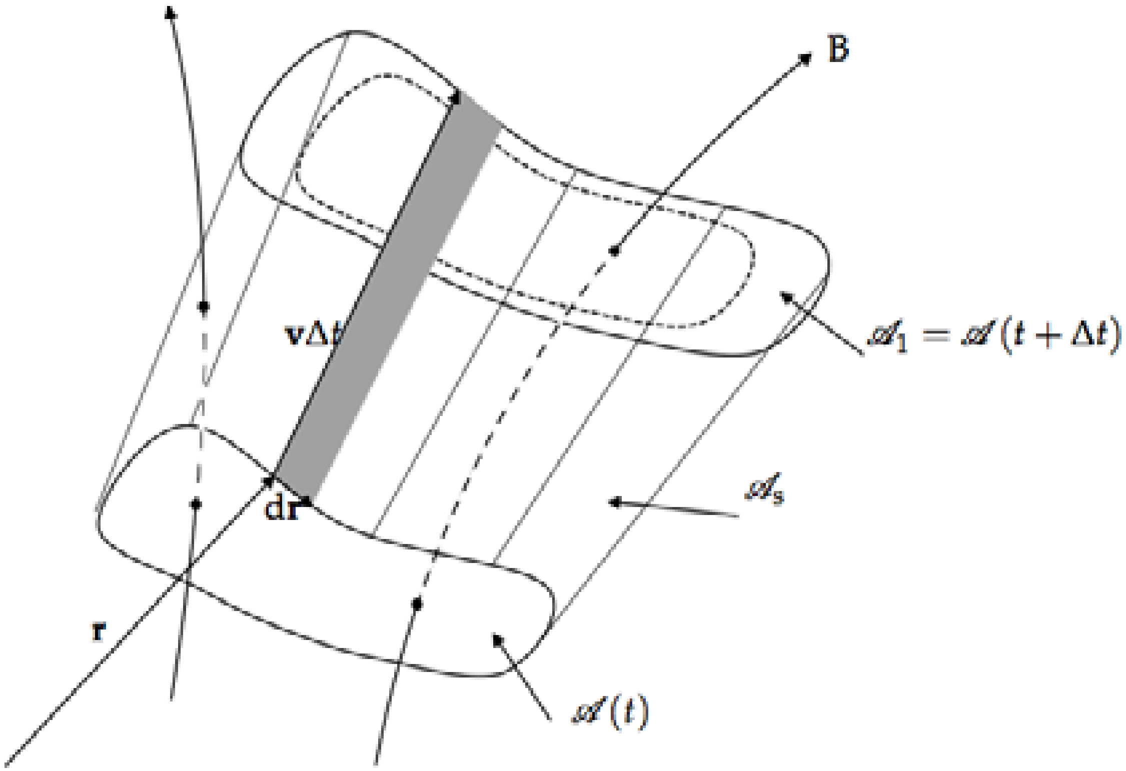


$$\int_{\partial\mathcal{A}} \mathbf{E}\cdot \mathrm{d}\mathbf{r} = -\int_{\mathcal{A}} \frac{\partial}{\partial t}\mathbf{B}\cdot \mathrm{d}\mathbf{a} = U_{12} + \int_{\mathcal{S}_1} \frac{1}{\varkappa a} I \,\mathrm{d}s + \int_{\mathcal{S}_2} \frac{1}{\varkappa a} I \,\mathrm{d}s - \int_{\mathcal{S}_2} (\mathbf{v}_{\mathrm{m}} \times \mathbf{B})\cdot \mathrm{d}\mathbf{r}$$

$$\int_{\mathcal{A}} \frac{\partial}{\partial t}\mathbf{B}\cdot \mathrm{d}\mathbf{a} = \int_{\mathcal{S}_2} (\mathbf{v}_{\mathrm{m}} \times \mathbf{B})\cdot \mathrm{d}\mathbf{r} \qquad U_{12} = -R\, I,$$

The Convective Derivative

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathscr{A}} \mathbf{B}\cdot\mathrm{d}\mathbf{a} = \int_{\mathscr{A}} \frac{\partial}{\partial t}\mathbf{B}\cdot\mathrm{d}\mathbf{a} - \int_{\partial\mathscr{A}} (\mathbf{v}\times\mathbf{B})\cdot\mathrm{d}\mathbf{r}\,.$$



The contour must change continuously!

$$U(\partial\mathscr{A}) = -\int_{\mathscr{A}} \frac{\partial}{\partial t}\mathbf{B}\cdot d\mathbf{a} + \int_{\partial\mathscr{A}} (\mathbf{v}_{\mathrm{p}} \times \mathbf{B}) \cdot d\mathbf{r}$$
$$= \int_{\partial\mathscr{A}} \mathbf{E}\cdot d\mathbf{r} + \int_{\partial\mathscr{A}} (\mathbf{v}_{\mathrm{p}} \times \mathbf{B}) \cdot d\mathbf{r}$$
$$= \int_{\partial\mathscr{A}} \mathbf{E}^{*}\cdot d\mathbf{r}$$

$$U(\partial\mathscr{A}) = -\frac{d\Phi}{dt}$$

$$U(\partial\mathscr{A}) = \int_{\partial\mathscr{A}} \mathbf{E}'\cdot d\mathbf{r} = \int_{\mathscr{A}} \operatorname{curl}\mathbf{E}'\cdot d\mathbf{a}$$

Proper frame EMF

$$= \int_{\mathscr{A}} \operatorname{curl}\mathbf{E}\cdot d\mathbf{a} + \int_{\mathscr{A}} \operatorname{curl}(\mathbf{v}_{\mathrm{m}} \times \mathbf{B}) \cdot d\mathbf{a}$$
$$= \int_{\mathscr{A}} \operatorname{curl}\mathbf{E}\cdot d\mathbf{a} + \int_{\partial\mathscr{A}} (\mathbf{v}_{\mathrm{m}} \times \mathbf{B}) \cdot d\mathbf{r}.$$

1) "The conductor must keep its material integrity"

The integration path has to be made of thin wire

2) "The integration path has to rest in the moving media"

Conclusion

- Faraday's law is always valid
- Some geometric constraints apply
- Calculating terminal voltages requires Ohm's law
- Lorentz invariance links the geometrical relations to real physics

- When slip rings or bulk material are present, always use the local Ohm's law (all physics is local). The symmetry of the device must allow to use "arbitrary" integration paths, otherwise a boundary value problem must be solved.
- The "naive" application of the "flux rule" works in cases where the loop is made of a thin wire.